Numerical Simulation and Visualization of Lava Flows

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Abstract

The study of the behavior of lava flows plays an important role in predicting, preventing and reducing the consequences of volcanic eruptions. Lava has been used as a building material for centuries and has been a source of nutrients for agriculture, but lava flows remain a threat to human activities.

Lava flow process is modelled as a spread of a viscous inhomogeneous incompressible fluid under the influence of gravitational forces. The mathematical model is described by the Navier-Stokes equation and the continuity equation with the corresponding initial and boundary conditions. The model takes into account the variable viscosity of the lava, which depends on the volume fraction of crystals.

As a spreading surface, we use the generated topography, which is a realistic slope of a mountainous area, formed taking into account natural geological processes. Numerical simulation is carried out using the meshless SPH method. The results of various cases of modeling of lava flows over the surface are presented.

Simulation results are visualized using our custom-developed Cinema Science 3D approach. It allows generating a custom 3D visualization with a simple CSV file configuration. We used it for presenting our results in a natural view, showing underlying terrain as mesh and lava as points, moving and changing according to time and other computation parameters. This view was enough for achieving visualization aims of our research.

Keywords: lava, numerical simulation, scientific visualization, smoothed-particle hydrodynamics, SPH.

1. Introduction

During volcanic eruptions, the spread of lava flows poses a significant risk to nearby communities, homes, and infrastructure. In effusive eruptions, a lava flow begins to form when molten rock erupts onto the Earth's surface and slowly spreads across the surface from a volcanic vent. Thus, various gravitational flows arise, depending on the chemical composition and temperature of igneous rocks, the quantitative content of crystals in the lava, and also on the topography of the surface along which the lava flows [1, 2]. Making timely lava flow forecasts is critical for volcano monitoring and emergency management during volcanic eruptions [3]. The core of the procedure is the creation of mathematical lava flow models for prediction of combination of lava flow direction, velocity, propagation, and duration of dangerous lava flows. Modern HPC hardware and software allows carrying out computer experiments to obtain a numerical solution of such mathematical models [4]. Although thermal effects play an important role in the flow of lava flows, simplified isothermal analytical and numerical models have demonstrated how lavas flow in the absence of cooling [2, 5].

A key requirement for computer code for lava flow simulation is that it must be able to provide timely forecasts in real time, that is, during a real natural process. At present, numerical simulation of viscous fluid flows is widely used in the study of volcanic processes and plays an important role in understanding the dynamics, morphology, and thermal evolution of lava flows [6]. In the past few decades, a new class of meshless SPH methods has been developed as an alternative to traditional mesh-based methods. The method considers a continuous medium discretization by a set of particles that move freely along the characteristics of the advection equation. Thus, it is natural to consider the equations of dynamics of a continuous medium in Lagrangian coordinates, and after discretization, partial differential equations for determining the speed, location, temperature, and other characteristics of the medium are transformed into ordinary differential equations for the evolution of each particle. The number of such differential equations corresponds to the number of particles in the discretization of the problem. SPH methods are used for mathematical modeling of lava flows due to the fact that this approach is able to describe a free surface flow, a multicomponent medium (namely, a multiphase gas-liquid-solid medium).

As a spreading surface, we use the generated topography, which is a realistic slope of a mountainous area, formed taking into account natural geological processes. Numerical simulation is carried out using the meshless SPH method. The results of several model cases of lava flows over the surface are presented.

2. Problem formulation

Mathematical model The process of spreading of a viscous inhomogeneous incompressible Newtonian fluid under gravitational force will be used to describe the lava flow motion. The mathematical model for such process is outlined by the Navier-Stokes equation and the continuity equation, which in Lagrangian coordinates have the form

$$\rho \frac{Du}{Dt} - div\big(\mu(grad\, u + (grad\, u)^T)\big) = -grad\, p + \rho g, \tag{1}$$

$$\frac{D\rho}{Dt} = -\rho\, div\, u = 0, \tag{2}$$

with the initial condition: $u\,(0, x) = u\,_0\,(x)$, where $u\,(t, x) = (u_1, u_2, u_3)$ is the velocity vector, $g = (0, 0, -9.81)$ is the acceleration vector due the gravitational force, $p(t, x)$ is the pressure, $\rho(t, x)$ is the lava density, $\mu(t, x)$ is the dynamic lava viscosity, t is the time, $x = x\,(t) = (u_1, u_2, u_3)$ is the characteristic of the advection equation

$$\frac{D\rho}{Dt} = 0, \quad \rho(t = 0, x) = \rho_0(x);$$

$grad$ and div are the gradient and divergence operators, respectively.

We assume that the viscosity of lava depends on the volume fraction of crystals [7, 8]

$$\mu(\varphi) = \mu_0\big(1 + \varphi^\delta\big)\left[1 - (1 - \xi)\cdot erf\left(\frac{\sqrt{\pi}}{2(1 - \xi)}\varphi(1 + \varphi^\gamma)\right)\right]^{-B\phi_*}, \tag{3}$$

where $\varphi = \phi/\phi_*$, $\phi \in [0, 1]$ is the volume fraction of crystals and ϕ_* is the specific value of the volume fraction of crystals; μ_0 is is the specific lava viscosity factor, B is the theoretical value of the Einstein coefficient (it has been experimentally established that the Einstein coefficient ranges from 1.5 to 5 [9]); $\delta = 7.24$, $\gamma = 5.76$ and $\xi = 4.63 \times 10^{-4}$; erf($\cdot$) is the error function.

Any melt consists of a liquid phase, gas and solid crystals, which tend to an equilibrium state. The volume fraction of crystals is determined from the evolution equation describing the simplified kinetics of the crystal growth during the crystallization due to magma degassing

$$\frac{D\phi}{Dt} = -\frac{\phi - \phi_{eq}}{\tau}, \tag{4}$$

with the initial condition $\phi(t = 0, x) = 0$. Here ϕ_{eq} is the volume fraction of crystals in the equilibrium state, which depends on the amount of the water dissolved in the magma and on

temperature; τ is the crystal growth time. The smaller τ, the faster the crystallization process converges to its equilibrium state.

As a spreading surface, we use the generated topography G (see the figure 1), which is a realistic slope of a mountainous area, formed considering natural geological processes. At this surface the viscous friction condition is satisfied:

$$u \cdot n = 0, f_G = \mu_G \frac{\partial u_{tan}}{\partial n}, G \subset R^3,$$

where u_{tan} is the projection of the velocity vector onto the tangent plane at the points of G, n is the normal vector to some surface. A volcanic crater (vent) with a diameter of 30 meters has been formed on this surface, in which a particle emitter is located, which generates particles with a constant boundary velocity $u = u_c$ and the volume fraction of crystals $\phi = \phi_c$.

The values of initial parameters of a particle that leaves a crater are presented in the table 1. These parameters correspond to the average values of eruptions with basaltic lavas.

Table 1. Initial parameters of a particle emerging from a crater

Parameter	Value	Dimension
Density	$\rho_0 = 2600$	$kg \cdot m^{-3}$
Boundary viscosity	$\mu_G = 10^{10}$	$Pa \cdot s$
Volume of cristal's fraction	$\phi_c = 0.4$	
Specific value of the volume fraction of crystals	$\phi_* = 0.384$	
Equilibrium state of the crystal	$\phi_{eq} = 0.83$	
Crystal growth time	$\tau = 3 \times 3600 \times 24$	sec
Velocity magnitude	$|\mathbf{u}_c| = 0.01$	$m \cdot s^{-1}$

SPH method of the model approximation In the SPH approach, a continuous medium is represented as a set of N particles interacting with each other under the application of various forces $\mathbf{F}$. According to (1), Newton's law is valid for each particle. The particle with the number i ($i = 1, ..., N$) at the moment of time t is located at the point $\mathbf{x}_i(t)$, moves at the velocity $\mathbf{v}_i(t)$ and has physical values, namely the pressure $p_i|$, mass m_i, density ρ_i, viscosity μ_i, volume fraction of crystals ϕ_i. The influence of a single particle on the properties of a liquid is estimated in accordance with the distance from it to the point of interest. Discrete particles have a characteristic radius $h > 0$ (sometimes this parameter is called the distance of influence of a particular particle on its neighbors). In the direction of increasing distance of influence, the properties of the particle are smoothed using a symmetric kernel function W. That is, the approximation of the value of a physical quantity A at the coordinate $\mathbf{x}(t)$ has the form:

$$A(x(t)) = \int_{R^3} A(y)W(|x - y|, h)dy.$$

Integration (1) by volume $V_i = m_i/\rho_i$ allows us to write Newton's second law for the i-th particle in the form

$$m_i \frac{Du_i}{Dt} = F_i^{visc} + F_i^{press} + F_i^{ext}, \quad i = 1, ..., N.$$

Mathematical justification and implementation of the SPH method are described in [10]. The software for this numerical experiment have been developed with SPlisHSPlasH, an open-source C++ library for the physical modeling of liquids. The main features of the package are described in [11]. To simulate lava dynamics, software was written with consideration of variable viscosity, also changes were made to the solver pressure calculation stage, and kinetics of crystal growth and viscosity have been added. The calculations were implemented on a personal computer running OS Ubuntu 14.10 without the use of a GPU. Parallelization is based on the OpenMP interface, which is designed for programming multithreaded applications on multiprocessor systems with shared memory.

Surface topography Several test scenes were generated to simulate lava flow. One of the scenes represents a section of a mountain surface created in the Wold Machine package [12]. This scene has sections of steep and gentle descents for expert assessment of the correctness of the flow behavior under various conditions, as well as formed active and inactive craters. Hydro- and wind- erosion parameters characteristic of Hawaiian-type volcanoes was used for formation. An example of such a surface is shown in figure 1.
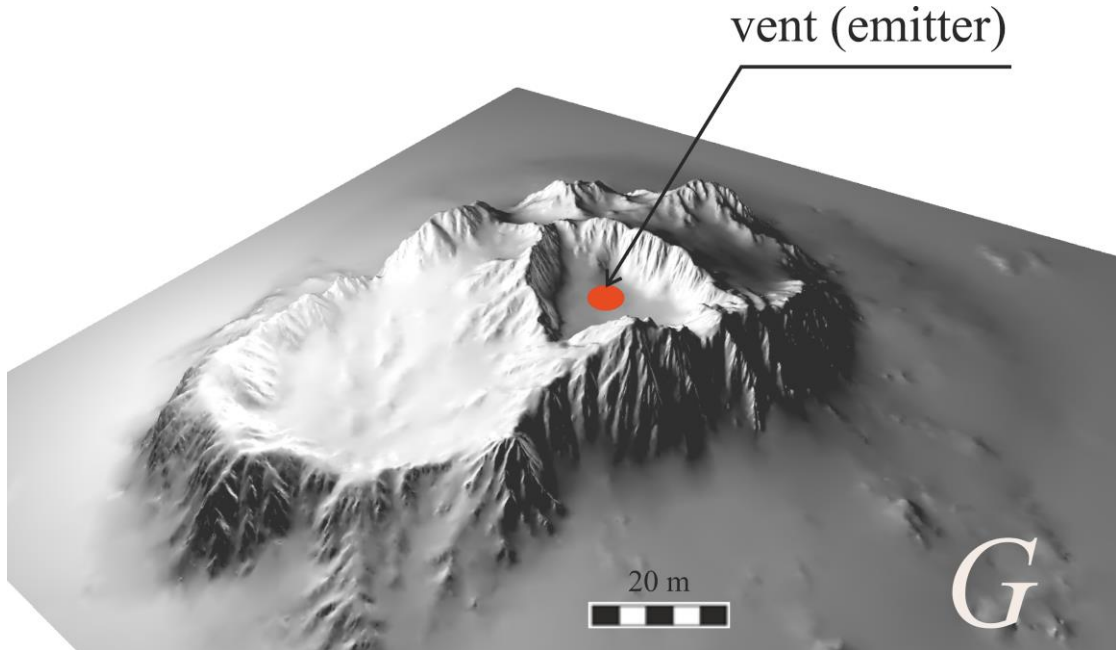

Figure 1: Artificial lava flow surface, generated considering the features inherent in Hawaiiantype volcanoes

3. Numerical Simulation Results

To organize a computational experiment, it is necessary to set the following initial data for calculations:
- Topography of the area over which lava flows will spread;
- Eruptive input conditions: volumetric velocity of lava extrusion, geometry of the vent from which lava will erupt;
- Boundary condition on topography and interaction of lava with the environment along its free boundary;
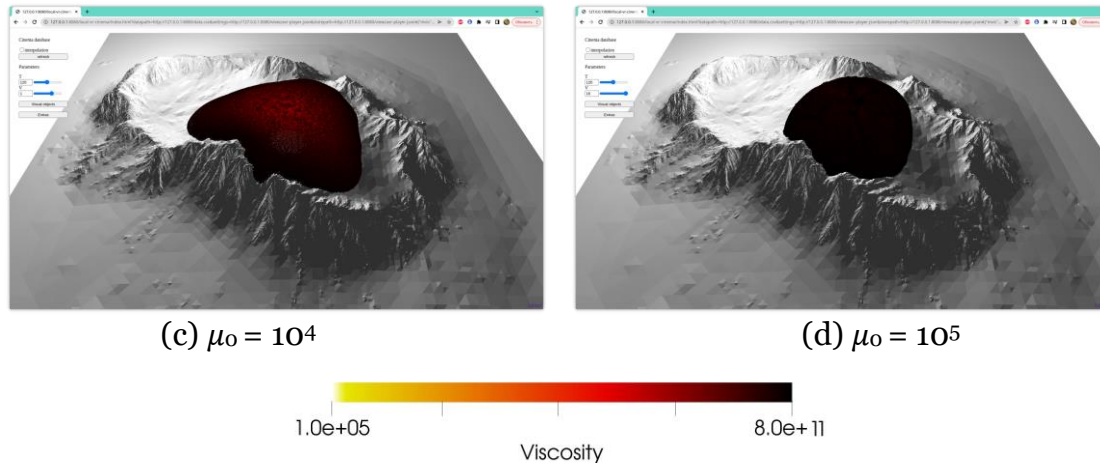- Physical properties of lava: the density, crystal contents, rheology.

Computational experiments were carried out for the specific lava viscosity factor $\mu_0 \in \{10^2, 10^3, 10^4, 10^5\} Pa\ s$ on the time interval $[0; 2.5 \times 10^6]$ sec. The figures 2 show results of numerical simulations.

(a) $\mu_0 = 10^2$ (b) $\mu_0 = 10^3$

(c) $\mu_0 = 10^4$ (d) $\mu_0 = 10^5$
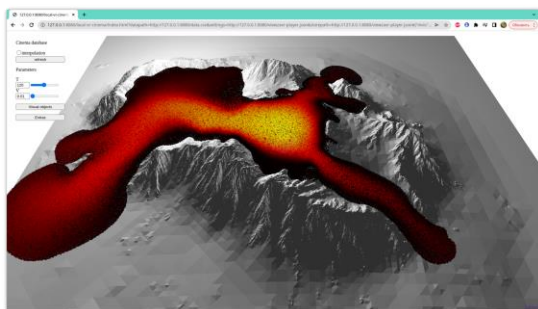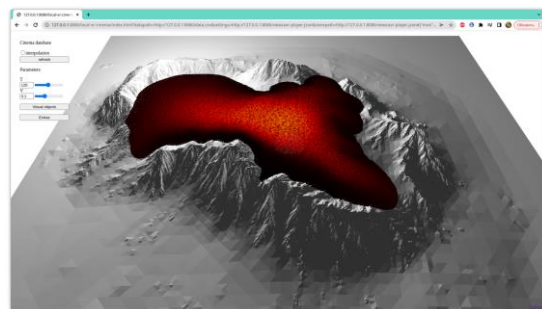
Figure 2: The figure shows the dynamics of lava spreading with dynamic viscosity depending on the level of crystals in the melt according to the formula (3) at the time moment 2.5×10^6 sec.

4. Visualization of results

4.1. Visualization task

In addition to the numerical calculations themselves, there was also the task of visualizing the results of the simulation. This task is relevant, since the visual method for evaluating the results of numerical modeling is very common in volcanology. The following aims were set for visualization:

- Show the dynamics of lava spreading over time;
- Within one visualization, show the results of several series of calculations with different initial parameters;
- Show the internal structure of the lava flow.

The first aim is of primary interest to an expert volcanologist, since it allows one to qualitatively assess the dynamics of spreading and the morphology of the resulting dome, and thus assess the overall "quality" of the simulation. The second aim is born from the frequent impossibility of specifying the exact values of the initial lava parameters. According to indirect data, a volcanologist can indicate parameters in a certain range. Therefore, a frequent request is the variation of some of the modeling parameters in one of the directions. The third aim is more 'cosmetic', as many rendering packages allow to slice object with a plane. To assess the dynamics of the distribution of some parameter (for example, the level of crystals or the distribution of relict particles) in space, a larger number of such slices may be required.

4.2. Visualization solution

An approach to 3D visualization developed at the Krasovskii Institute [13] has been chosen as the main visualization tool. It was based on CinemaScience [14]. It allows a custom 3D visualization to be programmed using simple CSV file [15]. In this file, which describes a table in a text format, three things are defined:

1. a list of parameters,
2. a list of visual artifacts, including their types,
3. a mapping between parameters values and artifact files.

Both parameters and visual artifacts are abstract to CinemaScience approach. The following model is used: when user selects some parameters values in graphical interface, a corresponding visual artifact files are loaded and presented in 3D space. Thus a wide range of visualizations may be programmed, including time-based.

To solve the first of two visualization aims defined above, the paradigm of describing the results of calculations in the CinemaScience format is perfect, since it allows to represent

each variable condition as a visualization parameter. For the stated visualization task, a CSV file of the following form was generated. An example of such a file is shown in the table 2.

Table 2. An example of the CSV file for the stated visualization task

T,	ViscoStep,	FILE_vtkpoints_lava,	FILE_obj_vulkan,	FILE_obj_emmiter
4,	10e2,	10e2/ParticleData_4.vtk,	vulk.obj,	emit.obj
5,	10e2,	10e2/ParticleData_5.vtk,	vulk.obj,	emit.obj
		...		
4,	10e3,	10e3/ParticleData_4.vtk,	vulk.obj,	emit.obj
5,	10e3,	10e3/ParticleData_5.vtk,	vulk.obj,	emit.obj
		...		
4,	10e4,	10e4/ParticleData_4.vtk,	vulk.obj,	emit.obj
		...		
1999,	10e5,	10e5/ParticleData_1999.vtk,	vulk.obj,	emit.obj

- There are two parameters defined, *T* and *ViscoStep*. While the first one is a simulation time step, the second one is the computational parameter representing viscosity. We put into one CSV file the results of series of computations with different *ViscoStep*.
- There are three visual objects defined, *FILE_vtkpoints_lava*, *FILE_obj_vulkan* and *FILE_obj_emitter*. The first one has 'type' *FILE_vtkpoints*, which corresponds to VTK file format [16]. The latter two have type *FILE_obj*, which corresponds to the OBJ file format.
- An input file for *FILE_vtkpoints_lava* visual object changes when parameters (*T* and *V iscoStep*) change. Input files for *FILE_obj_vulkan* and *FILE_obj_emitter* do not change.

We create a directory where we put the CSV file of the structure presented above, among with referenced data files. Then we run a visualization system which loads that file. From the first line of CSV file, it determines parameters and artifacts. Graphical controls are generated for all parameters. Their values, selected by a user, determine current input files for visual objects.

These visual objects are displayed in 3D scene, and user might change the position of the camera, zoom in and out, so on. Thus, user navigates both in geometry space using a camera and in parameter space by selecting a current combination of parameter values.

4.3. Discussion on visualization solution

As stated above, a user is free to select any available combination of parameter values. For example, a user may slide among some parameters (*T* or *ViscoStep*) while keeping other parameters constant.

The figure 2 shows different initial calculation parameters (variation of the initial viscosity parameter) for a fixed time parameter. On the contrary, the figure 3 shows the spreading dynamics at different moments of time for a fixed parameter μ_0.

(a) 1 hour;   (b) 8 hours;

(c) 25 hours;   (d) 36 hours;

1.0e+05   8.0e+11
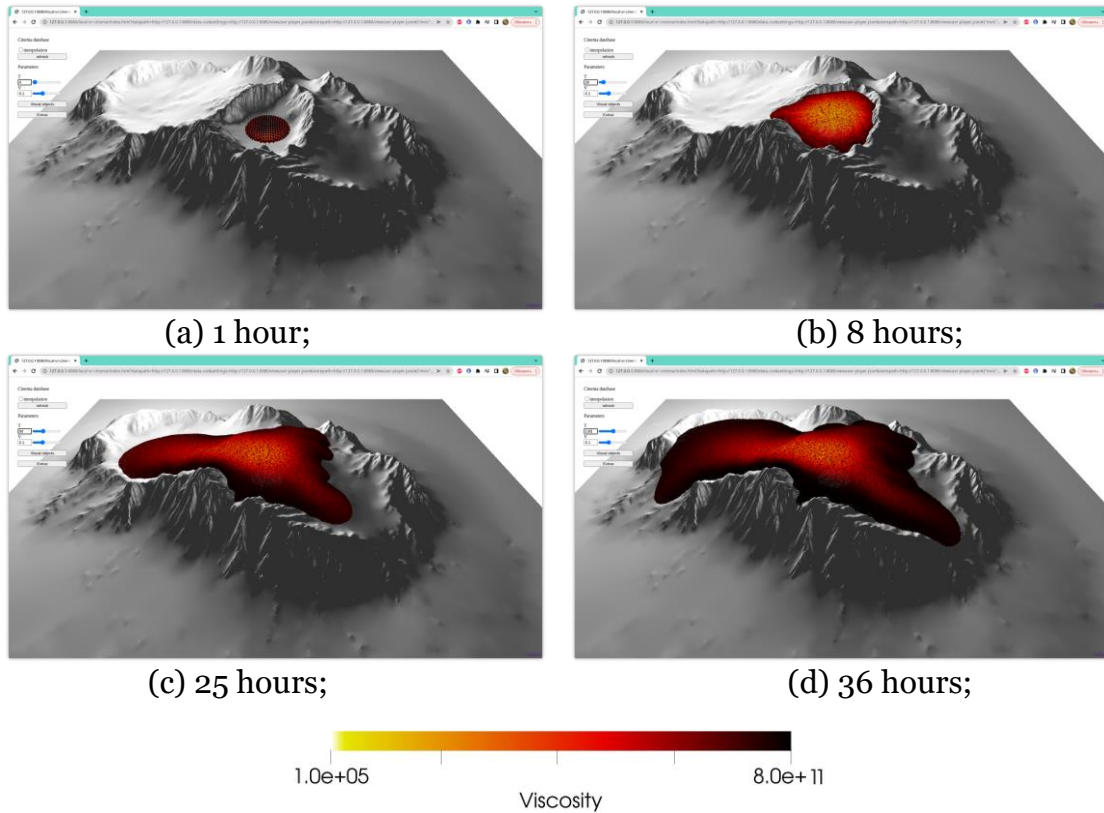
Viscosity

Figure 3: The dynamics of lava spreading with dynamic viscosity depending on the level of crystals in the melt according to the formula (3) with viscosity $\mu_0 = 10^3$ at different times

The last aim was achieved by using sets of "modifiers" – relatively simple scripts that allow to modify the behavior of the scene. The figure 4 shows the Clip serie modifier, which allows to make a regular set of sections parallel to the plane and with a given thickness. This makes it possible to evaluate the distribution of parameters within the flow over the entire volume, simultaneously.

Summarizing, using the CinemaScience 3D approach for scientific visualization, we got the following results:

Firstly, thanks to the simple description of the scene in CSV format, we are able to display

(a)   (b)
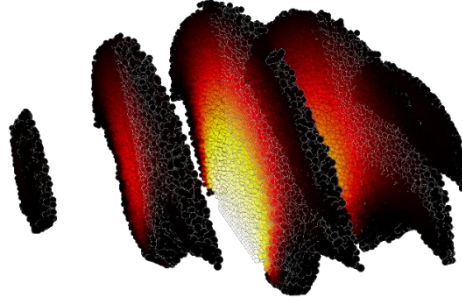
(c)

Figure 4: The figure shows a lava flow sliced with the Clip serie modifier. The color indicates the fraction of crystals, from 0.6 (yellow) to 0.8 (black). *(a)* clearly shows the formation of a "channel" of the flow, during which the lava adhering to the surface thickens and forms a trough. *(b)* shows how a solid crust is formed and a relatively liquid flow underneath, which can lead to the formation of lava "tubes" or cracking of the crust. *(c)* shows the formation of a lava dome with hard crust, which is formed during the outflow of highly viscous lava several computational results corresponding to the same model time in one scene. Thus, both static time slices and animated sequences can be displayed, that is very important for retrospective and comparative analysis. So, we can specify the calculation parameters as a visualization parameter and view the progress of the simulation process for different computational processes at one point in time.

Secondly, the description of the scene is not static and may change during the visualization process. This makes it possible to build the complex of online visualization. When new numerical results appear, new entries are added to the scene description. This allows the researcher to monitor the progress of the simulation and, if necessary, make changes to the model parameters or stop the calculations without waiting for the end of the numerical simulation. This is important because it can significantly save computing time. Another advantage of online visualization is the ability to process results directly from the supercomputer's storage, without the need to move all amounts of data.

And thirdly, the use of the modifiers system allows you to quickly create the necessary types of displays, including filters and additional entities that allow a full exploration of a visual object. So, the Clip serie modifier described above allows you to literally cut a point cloud into plates of the required thickness and with a convenient step, literally with one click, which allows you to see the internal structure of the flow.

The stated visualization task also might be solved in universal visualization systems, for example in Paraview [17]. However, we prefer CinemaScience-based approach. It allows defining scene dynamics using CSV file syntax and simple semantics. In a current task the required dynamics is simple. The corresponding state of the lava flow is shown by selecting T and *ViscoStep* value. In ongoing tasks, it may be more sophisticated. Even current dynamics is not easy to implement in ParaView. It allows importing series of files and play animations among them. Thus, to emulate our solution, user may manually open different groups of VTK files, each for different *ViscoStep*, and manage their visibility by turning them on and off. However, this is usable only for small amount of different *ViscoSteps*. To program more sophisticated dynamics, some kind of programmable filter will be required. It definitely requires more efforts than generating a CSV file. We believe the creation of CinemaScience 3D import plugin for Paraview can allow getting the best of two software solutions. Another problem of Paraview is an absence of required visual modifiers, exactly Clip series modifier. It can be implemented using programmable filter in Paraview. In our system we implemented this modifier using OpenGL shaders, adding it to user library of available modifiers.

5. Conclusion

A three-dimensional model of lava flow evolution was considered under the assumption that the lava viscosity depends only on the volume fraction of crystals, and this fraction, in turn, depends on the characteristic time of crystal growth. Lava flows are modeled using the SPH method. The visualization of the numerical results was performed using CinemaScience 3D approach. Using the visualization tool compatible with that approach, various slices of lava flows were examined. The distribution and formation of crystals were analyzed depending on the initial viscosity. This information is used for short-term and long-term forecasts of the eruption of real volcanoes by employees of volcanological observatories.

It is also necessary to add that, although the above results were obtained on synthetic data, they are as close as possible to real data. Now, numerical simulations are being carried out on real topologies obtained by satellite scanning.

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A. Online Resources

You can take a look at the ViewZavr project (which is an implementation of the described approach) and example scenes via

https://github.com/viewzavr/vr-cinema