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Numerical Calculation and Visualization of Pseudosymmetry Attributes of Biological Objects

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Abstract

The paper describes the approach and computer program for calculation and visualization of pseudosymmetry attributes of complexly structured objects of various nature. The approach is based on the group-theoretic and set-theoretic analysis. The program is aimed at the numerical estimation of bilateral pseudosymmetry of various nature objects both in 2D and in 3D case. In 2D case it calculates DoI (degree of invariance) of the object contour depending on the angle of its rotation relative to the axes of given coordinate system. The local maxima of DoI plot make it possible to determine the direction of bilateral pseudosymmetry axes of the object contour. In 3D case, the output data of DoI is a table of numerical values depending on zenith and azimuth angles of the spherical coordinate system. This table allows obtaining the set of pseudosymmetry planes of an object. There are some examples in the paper demonstrating the efficiency of the described approach and program.

Keywords: pseudosymmetry, DoI, invariance degree, Group theory, HTML5/Javascript languages.

1. Introduction

Symmetry is the property of remaining invariant under certain changes (as of orientation in space, of the sign of electric charge, of parity, or of the direction of time flow)—related to physical phenomena and of equations describing them [1].

Symmetry is observed everywhere in wild life, but biological objects, as a rule, are not perfectly symmetrical, but approximately only. Therefore, they are also called pseudosymmetric. Nevertheless, a living object can be characterized by a certain type of symmetry, along with other morphological, physiological, genetic and other features, according to which we distinguish one organism from another [2,3].

One of the fundamental problems of biology is to distinguish between the variability of shapes and sizes of morphological objects. In biology, there is a geometric continuum vision of organisms as integral forms. The laws of physique of organisms are embodied not only in external form, but also in structural elements: organs, cells, macromolecules, which are called biomorphs in biology. Moreover, each of the biomorphs is not only endowed with its own symmetry, but is also connected by a symmetry relationship with other biomorphs. This vision of biomorph represents the traditions of geometry in biology, and the corresponding section of biology called biosymmetry [4]. Symmetry should be considered as an immanent dualistic characteristic of a biological object, which inevitably manifests itself in ontogenesis. Methods for its calculation were developed as a special analytical tool for solving the fundamental problem of biological research - the distinction between the variability of the shape and size of biological objects. If the methodological significance of symmetry in biology and ecology is beyond doubt, then the methods for quantitative assessment of symmetry degree of biological objects remain a field for discussion [2], [4].

2. Numerical assessment of object DoI

Consider the problem of invariance degree of a finite object determining with respect to various isometric transformations from the set theory point of view. Let us describe such an object by a function of three variables $f(x_1, x_2, x_3)$ integrable in a limited range of its arguments. Then we can take into consideration the invariance degree of this function with respect to some operator g' of x_1, x_2, x_3 coordinate transformation. The value that gives the numerical value of the invariance degree with respect to the coordinate transformation must be a number that is assigned to each function, i.e., should be functional. In this case, the functional should change from –1 to +1. The value +1 should correspond to the case when the function $f(x_1, x_2, x_3)$ is completely invariant (symmetric) with respect to the given operation g', the value -1 should correspond to the case when the function $f(x_1, x_2, x_3)$ is completely antisymmetric with respect to the operation g'. Thus, the invariance degree of the real function $f(x_1, x_2, x_3)$ with respect to some transformation g', we mean the functional $\eta_{g'}$ in the form of the following convolution (see work [2])

$$\eta_{g'}[f(x_1,x_2,x_3)] = \frac{\int\limits_{\Omega} f(x_1,x_2,x_3) f(g'(x_1,x_2,x_3)) dx_1 dx_2 dx_3}{\int\limits_{\Omega} f(x_1,x_2,x_3)^2 dx_1 dx_2 dx_3}, \qquad (1)$$

Here we integrate over the entire area Ω where function $f(x_1, x_2, x_3)$ is defined. This convolution depends on the type of function $f(x_1, x_2, x_3)$, and on the type of operation g'. If the arguments domain is the entire set of real numbers and the function is a constant, then $\eta_{g'}[f(x_1,x_2,x_3)]=1$. If the operation g' is a single transformation, then $\eta_{g'}[f(x_1,x_2,x_3)]=1$ as well, regardless of the form of the function and the scope of its arguments.

Let us take into account the case when function $f(x_1, x_2, x_3)$ is not a constant or where the domain of its arguments differs from the set of all real numbers. If $f(x_1, x_2, x_3)$ is invariant completely with the argument's transformation, which is described by the operator g', then $f(x_1,x_2,x_3)= f(g'(x_1,x_2,x_3))$ and, therefore $\eta_{g'}[f(x_1,x_2,x_3)]=1$. If $f(x_1, x_2, x_3)$ is antisymmetric with respect to the given transformation, then $\eta_{g'}[f(x_1,x_2,x_3)]=-1$. If $\eta_{g'}[f(x_1,x_2,x_3)]=0$, then we can assume that the function is completely asymmetric with respect to transformation g'. In fact, this means that there is no single corresponding element of the structure under consideration both on one side and on the other side with respect to the symmetry agent (plane, axis, etc.)

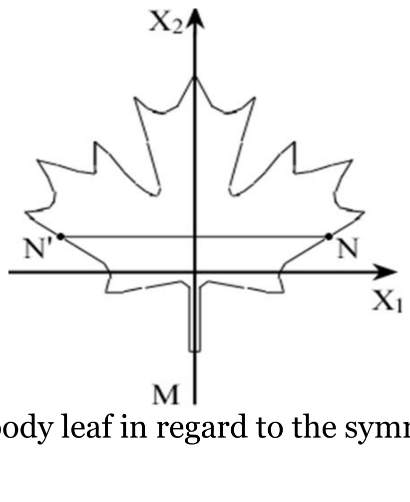

Figure 1. Woody leaf in regard to the symmetry axis M

Let us describe the application of convolution (1) to determine the 2D object degree of symmetry. As an example, consider a bilaterally symmetrical object, such as a leaf (see Fig.1). Let us introduce the function $f(x_1, x_2)$, which is equal to 1 at the points located on the surface of the leaf, and is equal to zero at all other points. We choose the axes of the orthogonal coordinate system so that the X_1 axis is perpendicular to the symmetry axis M and the origin lies on this axis (Fig.1). Then the bilateral operation in M axis transforms a point *N* with coordinates (x_1, x_2) into a point *N'* with coordinates $(-x_1, x_2)$. In our case of an absolutely symmetric leaf the values of the function at these points are non-negative and equal to each other

$$f(x_1, x_2) = f(-x_1, x_2). \quad (2)$$

Then the integral $\int_\Omega f(x_1, x_2) f(-x_1, x_2) ds$ over all points of the leaf surface Ω is numerically equal to its area. The same value is equal to the integral in the denominator of expression (1) $\int_\Omega f(x_1, x_2)^2 ds$, so that for an absolutely symmetrical leaf we get $\eta_{g'}[f(x_1, x_2] = 1$.

In the vast majority of 2D cases, when complex objects are not completely symmetrical, it is impossible to calculate the integrals analytically. Therefore, the image of the object is preliminary rasterized and replaced by a set of pixels. In this case we can take into account the set of pixels at the contour of the object image and modify the convolution relation by the function $f(x_1, x_2)$ as follows. Let the value of the *j*-th attribute of a given object be equal to *Ljr* to the left of the axis *M* in some *r*-th region (cell) of the partition, and *Rjr* in the symmetrically located region to the right of *M*, respectively. Then the degree of invariance of the object according to the *j*-th attribute relative to the axis *M* will be written in the form [3, 5, 6]

$$\eta_{g'}[M] = \frac{2\sum_{r=1}^{m} L_{jr} \cdot R_{jr}}{\sum_{r=1}^{m} (L_{jr}^2 + R_{jr}^2)}, \quad (3)$$

Here *m* is the number of pixels in the object rasterization to the left and right of the *M* axis.

The described group-theoretic approach allows us to create the program for the numerical estimation of the pseudosymmetry of various nature objects both in 2D and in 3D case.

In 3D case, the input data is an object geometry file in STL format. This geometry can be obtained either by generating geometry by any 3D geometric modeling system, or by 3D scanning of real objects with subsequent data conversion to STL format. The output data of invariance degree are a 3D table containing the calculated values depending on zenith and azimuth angles of the spherical coordinate system. This table allows obtaining the set of pseudosymmetry planes of an object.

3. DoI of 2D objects

The input data in 2D case is an array of coordinates of the vertices of a closed polyline representing the contour of an object. The program initially decomposes the given polyline into a raster, and then calculates the invariance degree (Fig.2). The invariance degree is calculated by formula (3), where *Ljr* and *Rjr* are the raster coordinates *X* of the *Left* and *Right* pixels in Fig. 2.

Further, the polyline is rotated by a given angle relative to the coordinate system and the process is repeated anew. The program calculates the invariance degree of the object contour depending on the angle of its rotation relative to the axes of the given coordinate system. The value of the invariance degree varies from 0 to 1 depending on the angle of rotation. Value 1 corresponds to exact symmetry, 0 means no symmetry at all. The local maxima of the invariance degree make it possible to determine the direction of the pseudosymmetry axes of the object contour.

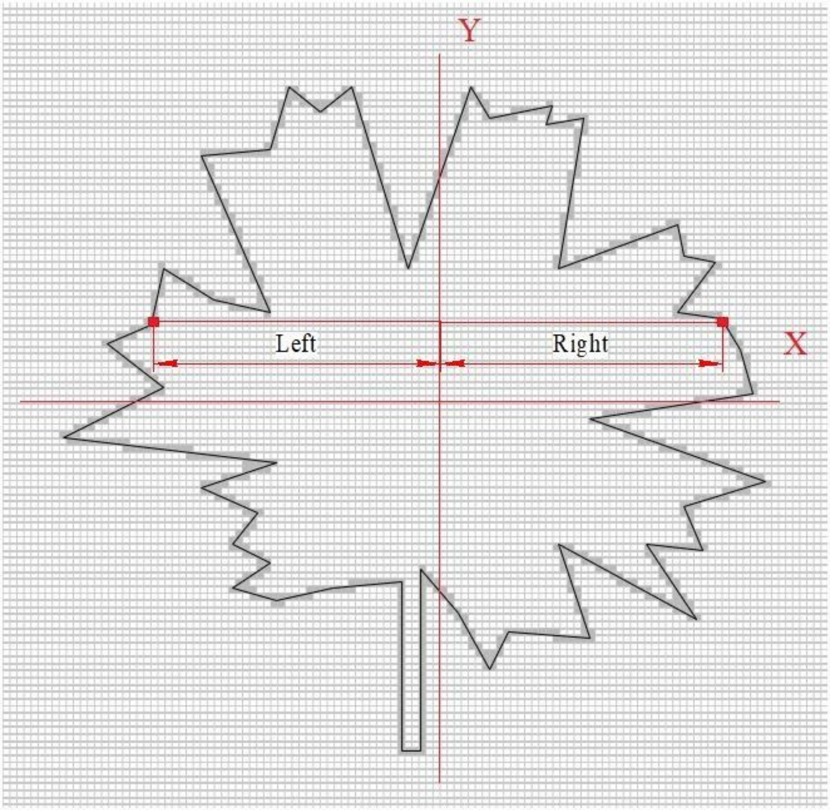

Figure 2. Scheme to apply formula (3) in 2D case

As a test example we consider the problem of determining the axes of invariance of a tree leaf (Fig.2). The leaf has an artificial shape, built in K3 CAD system.

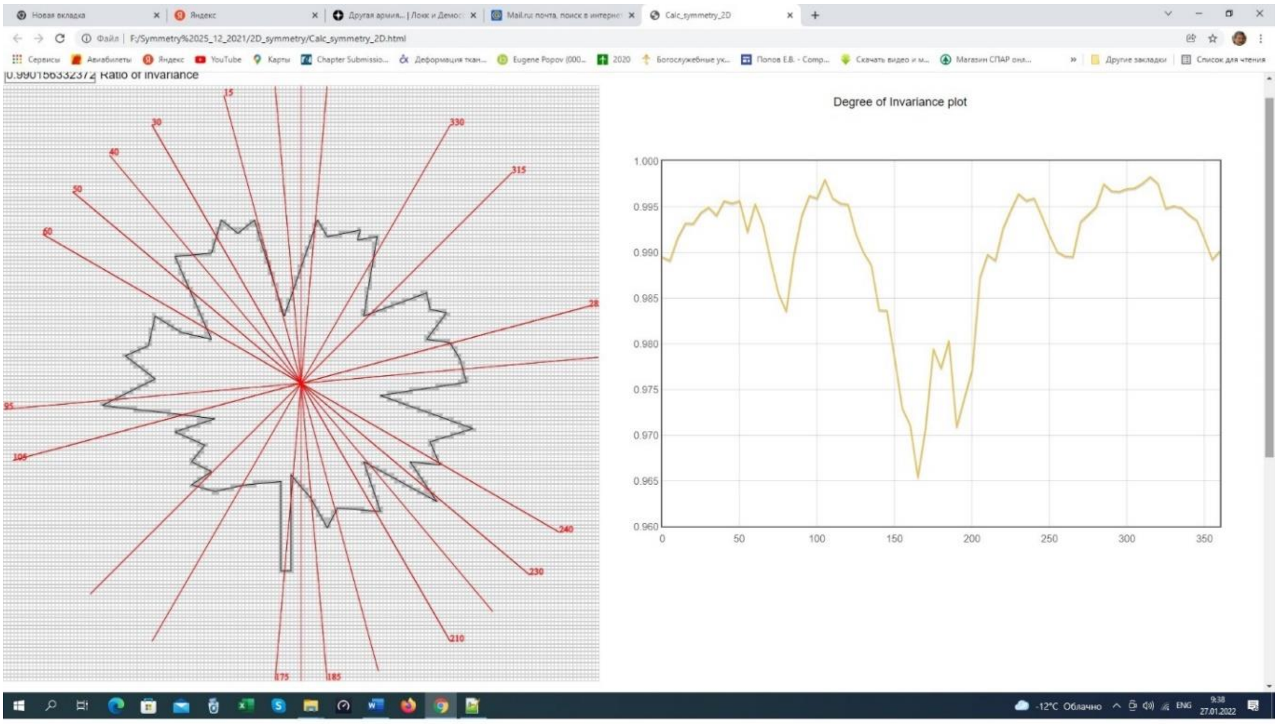

Figure 3. Test sample with invariance plot and pseudosymmetry axes

Fig.3 shows a plot of the leaf invariance degree calculated by formula (2), depending on its rotation angle relative to the coordinate system. The left part of Fig.3 shows the axes of invariance corresponding to all local maxima in the graph on the right.

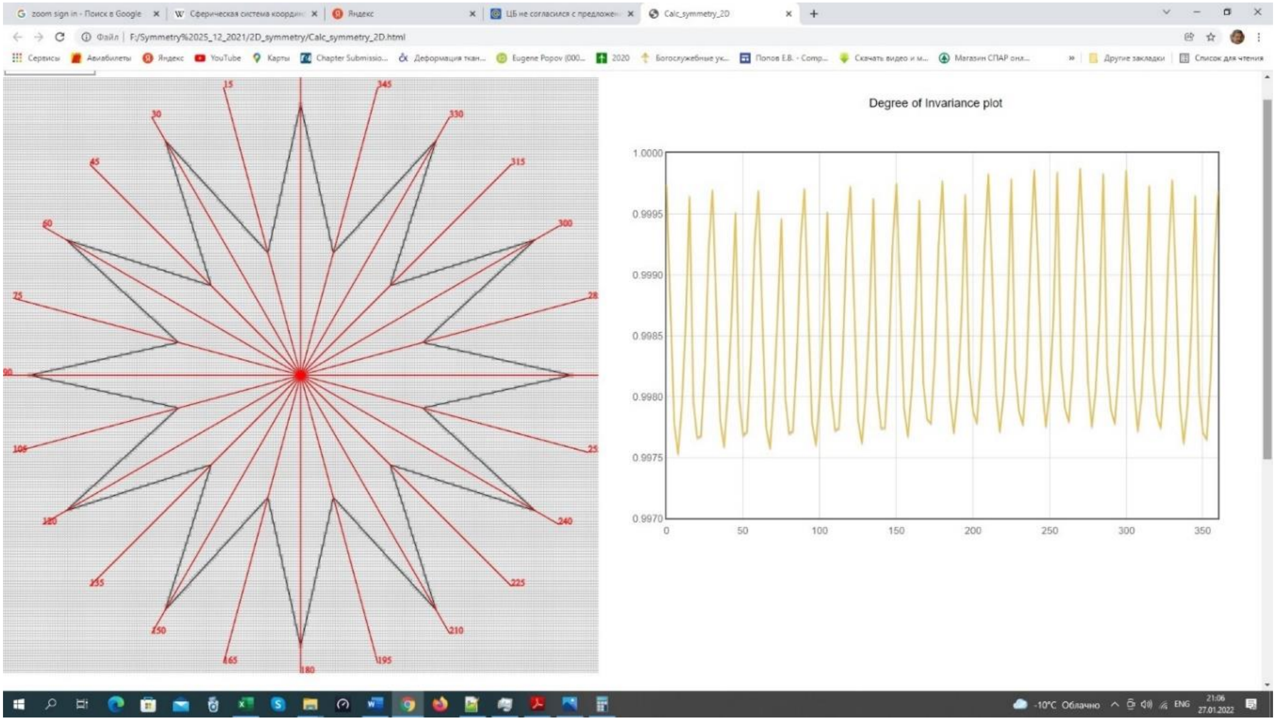

Figure 4. The bilateral pseudosymmetry axes of a 12-ray star

The following example refers to the calculation of the invariance degrees of a 12-ray star, which is a regular geometric figure. Fig.4 shows the axes of bilateral pseudosymmetry according to local maxima in the graph on the left side of Fig.4. As can be seen in Fig.4, there are 24 such maxima, the bilateral symmetry axes pass both along the tops of the rays of the star and along their troughs.

This obvious example fully corresponds to a similar example in the work [4, 6] and confirms applicability of the developed approach to estimation of pseudosymmetry axes position in 2D.

Another example concerns the evaluation of symmetry degree of the zygomorphic flower corolla (Fig. 5). The symmetry axes position of this figure is studied in detail in works [4] and [7]. Moreover, in work [4] one axis of symmetry was found, and in work [4] - five. In our study, we found 12 axes of symmetry, among which five coincide with the axes in [4], i.e., axes at 70°, 145°, 185°, 215°, 295°. The raster density in our case was equal to 180 × 180, and the pitch of the contour rotation angles was 10°.

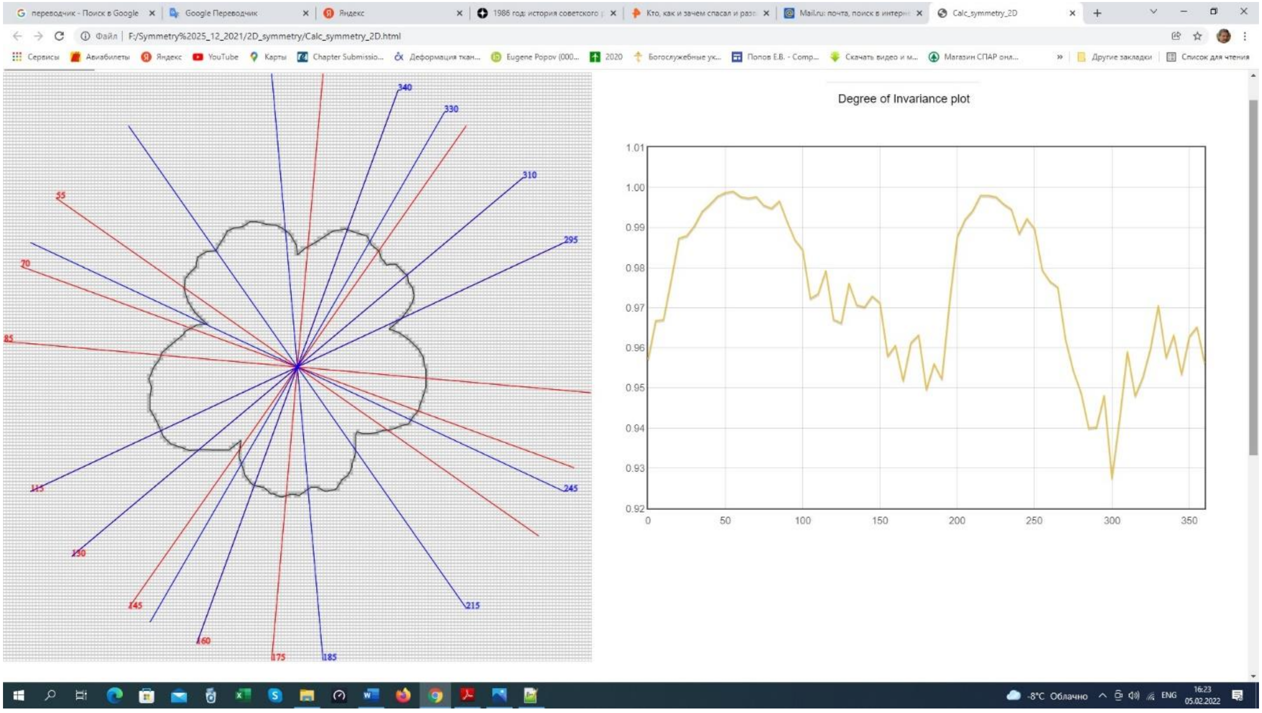

Figure 5. The corolla of a zygomorphic flower pseudosymmetry

4. DoI calculation of 3D objects

The study of the pseudosymmetry of a 3D object using computer technology requires the presence of a model of this object in the computer memory, obtained in one way or another. One way to obtain 3D object model is to develop it by any CAD system. The main disadvantage of this way is the requirement of good user skills in geometric modeling and the ability to create object models as accurately as possible. Another way to create 3D models is scanning. In this case, there is a real opportunity to obtain the most accurate and adequate model of a natural or man-made object. The output information of 3D scanning is a cloud of points in the coordinate system of the scanner working space, belonging to the surface of a real object. The standard software of most 3D scanners provides the transformation of this point cloud into a surface model, represented as a triangle grid that can be output in STL format.

For instance, Figs. 6 and 7 show scanned models of two biological objects, namely the starfish *Crossaster papposus* and skull of *Ondatra zibethicus*. We used these models as samples of our calculation.

Work [4] describes the program Symmetry 3D, which allows obtaining initial 3D surface of an object and calculating its pseudosymmetry. When using this program, the main problem is the object representation in the form of a set of 3D elements - *voxels*, which (according to the authors' opinion) is convenient for studying the symmetrical characteristics of 3D object. However, it should be noted that the space decomposition into a 3D raster and the voxelization of an object leads to the fact that the pseudosymmetry calculation becomes unreasonably expensive in terms of computer resources. The fact is that 3D raster with poor density can lead to the situation when the characteristics of a 3D object with complex shape will be determined too roughly. For example, it is recommended to use a cubic volume, the size of which lies in the range 50–1000, for example, 100×100×100. An attempt to refine the 3D raster density will lead to an increase in computational costs in a cubic dependence and may make the result unattainable in a reasonable time and at a reasonable cost under Windows XP/Vista/7/8/8.1/10 (64-bit). This circumstance forces the authors of the program to use parallel computing technologies with multi-core systems and specialized graphics cards.

In this study we developed a new approach based on an original object 3D model represented in STL form directly. The approach also applies expression (1) and its modification (3) to the numerical DoI (degree of invariance) calculation of 3D object for an arbitrary isometric transformation.

The main idea here is to extend the approach to applying relation (3) according to the scheme in Fig. 2 for 2D case to 3D case. However, it should be borne in mind that in the 3D case we have abandoned the 3D raster concept.

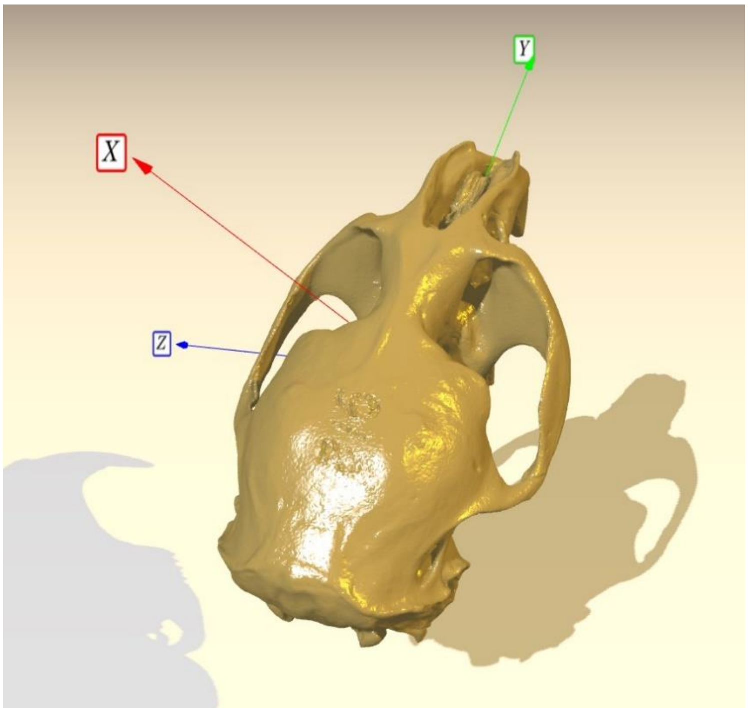

Figure 6. 3D model of starfish *Crossaster papposus*

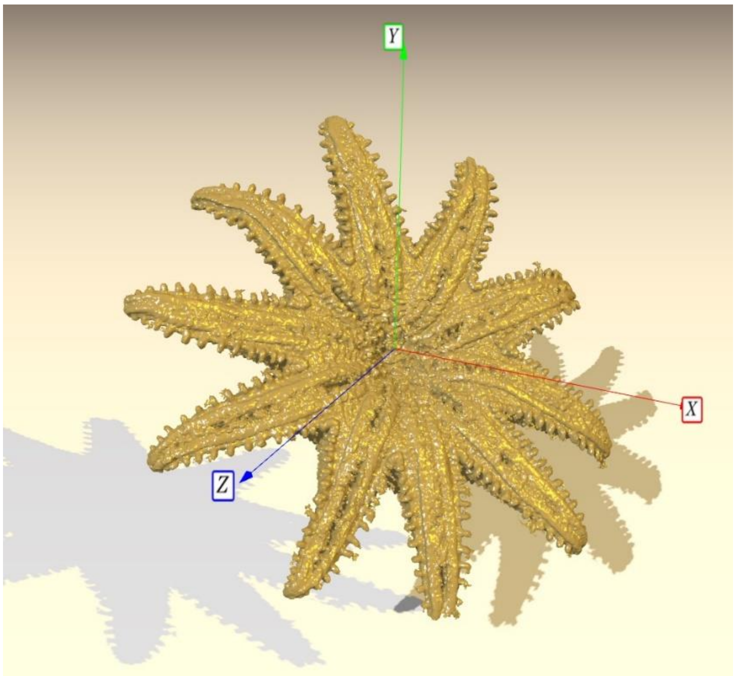

Figure 7. 3D model of *Ondatra zibethicus* skull

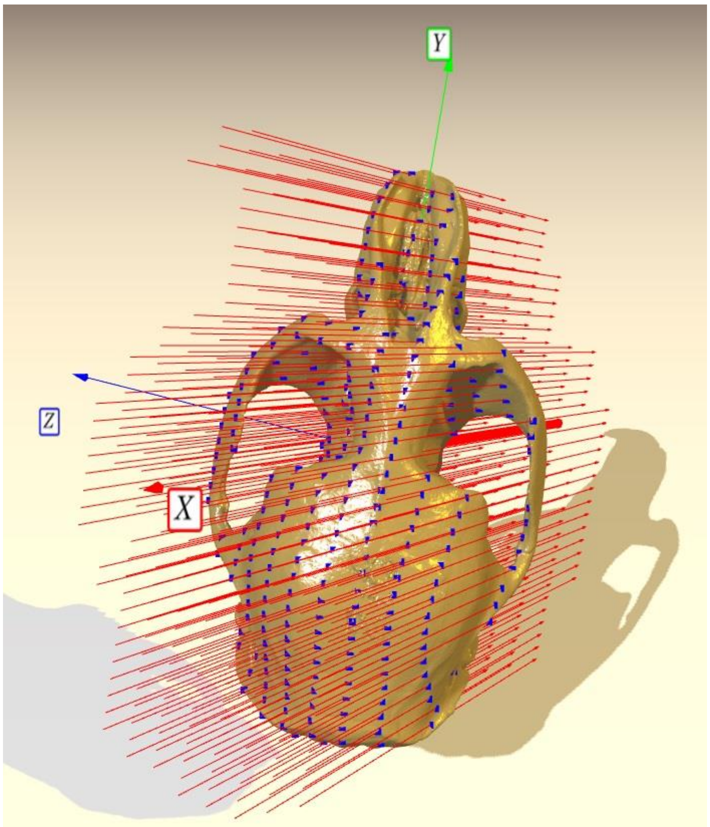

Figure 8. Scheme to apply formula (3) in 3D case

We replaced the raster with a package of lines parallel to each other and to X axis in such a way that when they cross the object, they form a set of intersection points (see Fig.8), the coordinates at X axis of which we then insert into formula (3). Obviously, in this case, the DoI is relative to the coordinate plane YZ. The accuracy of DoI calculation depends on the number of points, which, in its turn, hinges on the number of straight lines. The higher the line density, the higher the DoI accuracy. We studied the effect of line density for this accuracy, which showed that the number of lines of 40 × 40 relative to the YZ plane is quite sufficient for most calculations. An increase in the density of the line packet does not lead to a noticeable increase in the accuracy of DoI, but increases the calculation time according to a quadratic law.

The program calculates DoI at different orientations of the model relative to the coordinate system. The model cyclic rotation with respect to two zenith and azimuth angles of the spherical coordinate system is organized (angles α and β, see Fig.9) with unchanged position and direction of auxiliary straight lines (Fig.8). The output is a table of numerical DoI values at zenith and azimuth angles.

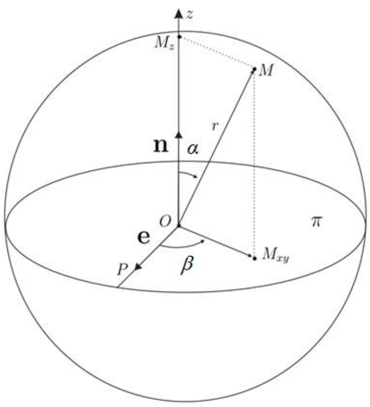

Figure 9. Spherical co-ordinate system

When calculating DoI the following parameters were given: the changing steps zenith α and azimuth β angles were taken to be the same and equal to 15°. The auxiliary straight-line density was limited in value 40 × 40.

The local maxima finding is a consistent comparison of the DoI values at each table element with the DoI values at the neighboring nodes surrounding it. A similar approach was described in work [8]. In our case there are 8 such neighboring nodes. If this value is the largest one, then one can consider this value as a local maximum. According to the output table we can build a plot in the form of NURBS surface represented in Figs. 10 and 11 for both our models where one can also see the position of local maxima as red points.

The local DOI maxima corresponds to the positions of the pseudosymmetry planes of each of the two objects. For example, Fig. 12 shows all the planes found for the muskrat skull. Figs.13 and 14 show the planes of pseudosymmetry of objects, corresponding to DoI *maximum maximorum*. We call them general bilateral symmetry planes. DoI values are given on these planes as textures.

In both cases, the found planes of bilateral pseudosymmetry quite obviously confirm the high efficiency of the developed approach to calculation of the DOI as in the case of 2D, and especially in the case of 3D

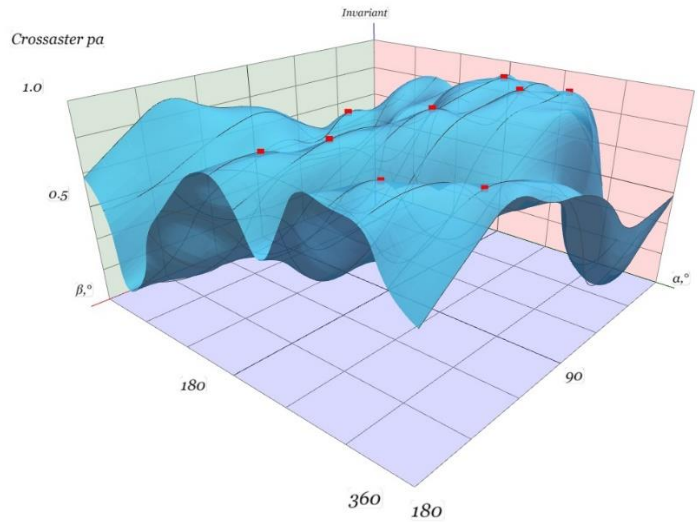

Figure 10. DoI surface of skull in unfolded spherical system with local maxima

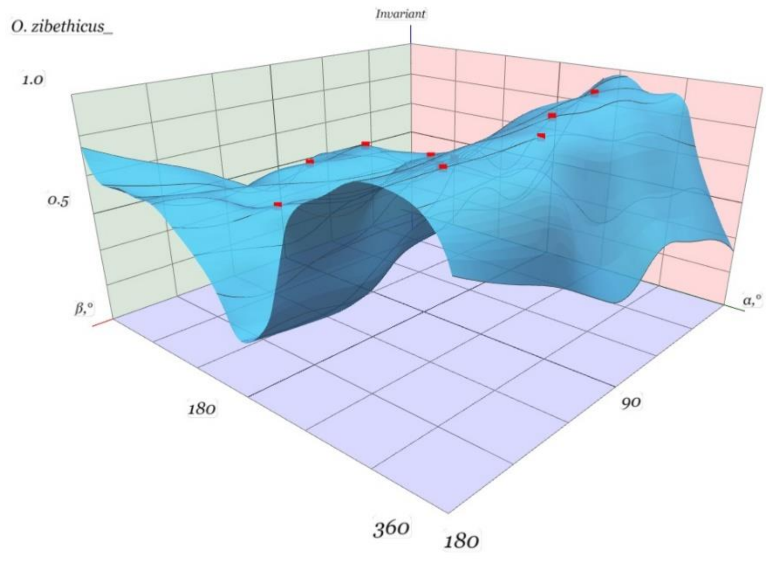

Figure 11. DoI surface of starfish in unfolded spherical system with local maxima

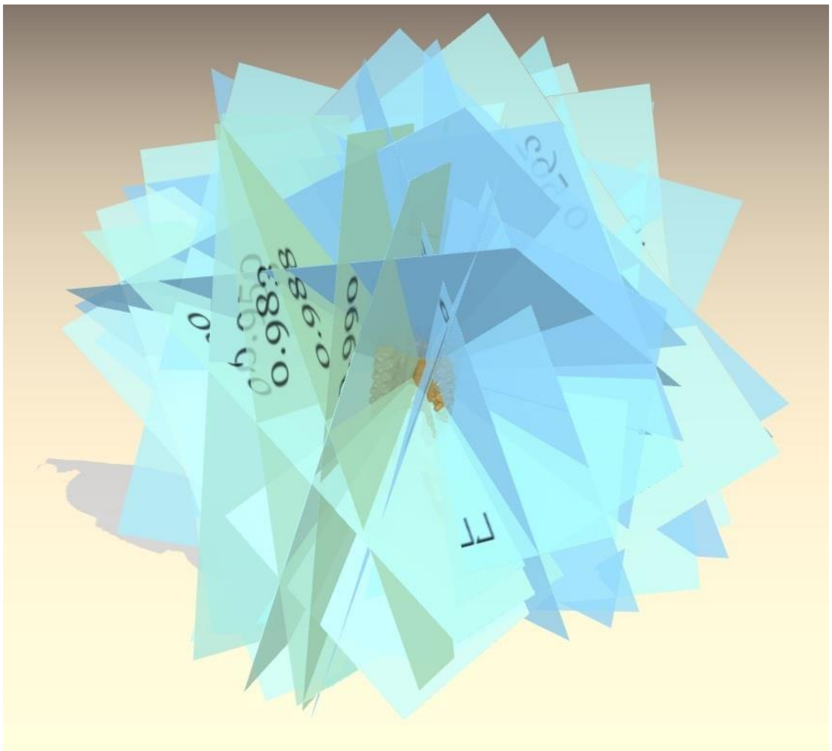

Figure 12. The entire set of local pseudosymmetry planes for the skull

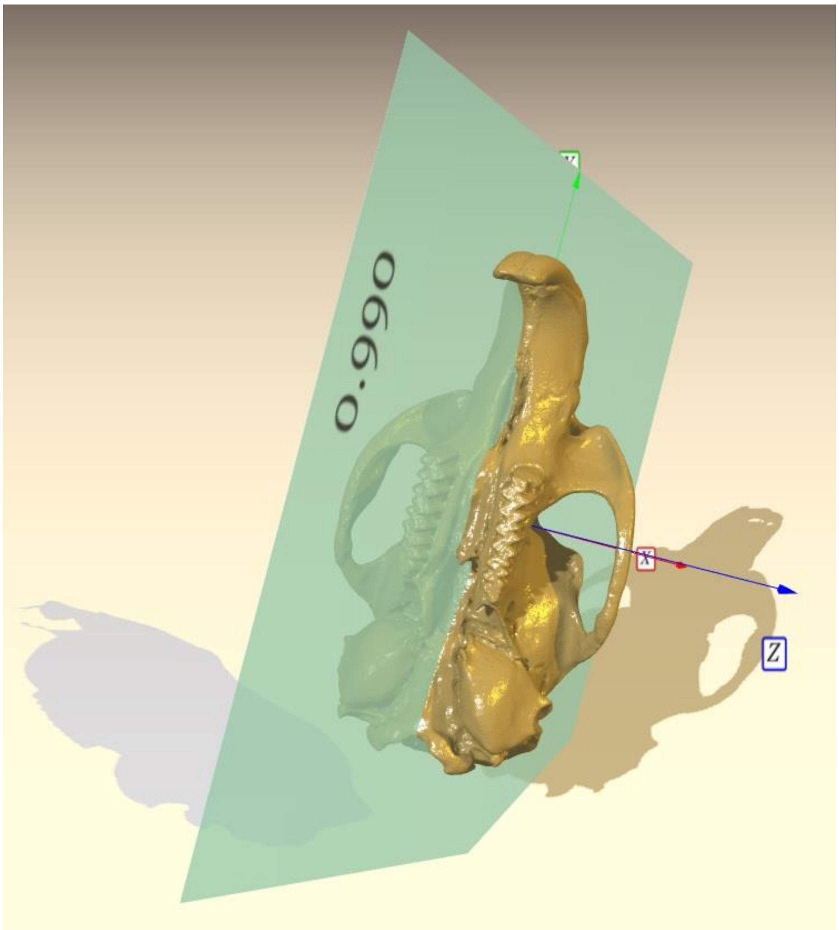

Figure 13. General bilateral symmetry plane of skull

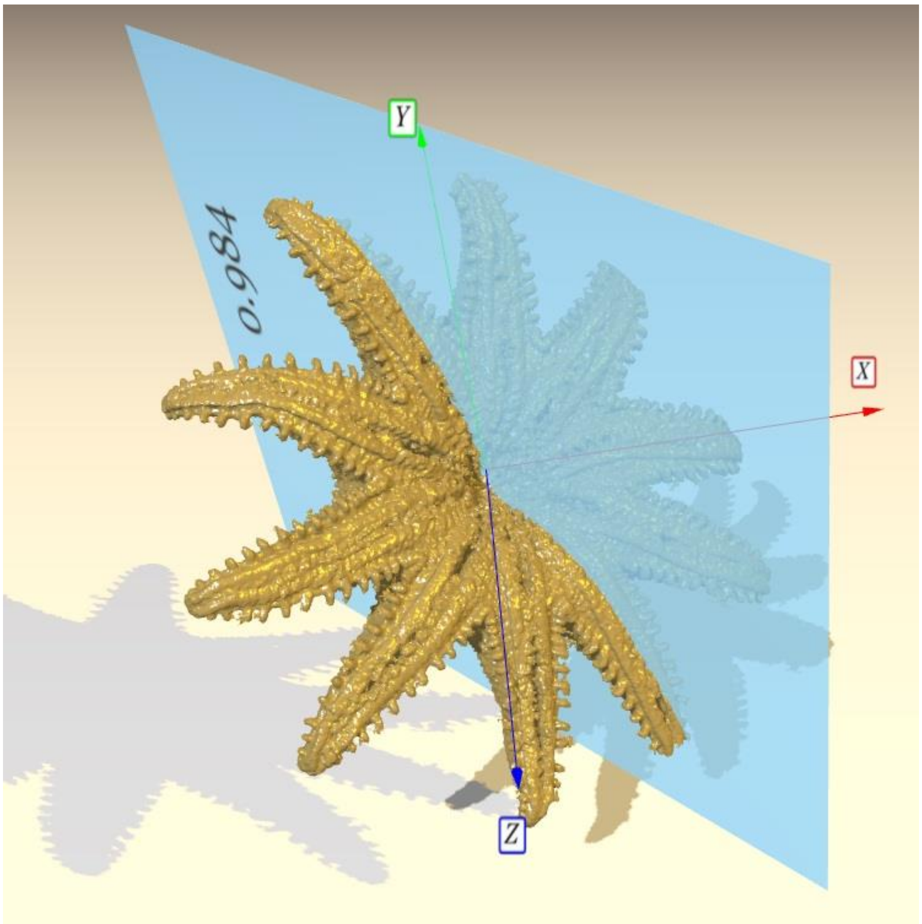

Figure 14. General bilateral symmetry plane of starfish

5. Computer program

We have created the program for the numerical estimation of the pseudosymmetry of various nature objects both in 2D and in 3D case on the basis of JavaScript language and CSS introduced to HTML5 code. We have chosen such a platform guided by considerations of ease coding and the program platform independence. In 2D case the program uses two libraries in addition to its original code, namely: jQuery.js as a tool to simplify the implementation of common procedures and plot.js - graph drawing library. In 3D case the original program code uses the capabilities of THREE.js graphic library. In addition to computing capabilities this provided the program with high quality photorealistic graphics. After the final calculations, the user has the opportunity to study the location of the pseudosymmetry planes in accordance with each found local minimum of the original functional.

6. Conclusion

Thus, the group-theoretic approach to the evaluation of object symmetry attributes is highly efficient and reliable. This has been proven by numerous studies in the field [3, 4, 5, 6, 9, 10]. However, the rasterization of an object model, which is traditionally applied with this approach, inevitably leads to unjustified expenditures of computer resources and computation time. This problem is greatly exacerbated in 3D case.

We devoted this article to the study of object bilateral symmetry with respect to the axis in 2D and relative to the plane in 3D. In order to reduce the calculation time and to save computer resources, we have developed a numerical approach to DoI calculation of objects in 2D and 3D. In 2D case we solve the problem by usual rasterization of an object in pixel space. This task does not require significant computer costs for the solution.

The main idea in 3D is the intersection of a 3D object by a package of parallel lines and calculation of their intersection points with the surface of an object. The set of obtained points is the basis for obtaining and minimizing the convolution functional in order to determine its local maxima. The *maximum maximorum* of the functional corresponds to the position of the bilateral pseudosymmetry plane. Such a technique made it possible to obtain a significant gain in saving computer resources and time while maintaining the accuracy of 3D problem solution. This is confirmed by solving some examples.

Further research will be devoted to solving the DoI problem in cyclic, dihedral, rotational and other types of pseudosymmetry.

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