Scientific Visualization, 2025, volume 17, number 4, pages 99 - 111, DOI: 10.26583/sv.17.4.10

This paper is an extended version of a contribution presented
at the Graphicon 2025 conference.

A Digital Twin Based on Mathematical Modeling of a Wind Turbine Unit

A.E. Bondarev[1]

Keldysh Institute of Applied Mathematics RAS, Moscow, Russia

[1] ORCID: 0000-0003-3681-5212, bond@keldysh.ru

Abstract

This article presents a comprehensive approach to creating and visualizing a digital twin of a wind turbine assembly. The study focuses on integrating mathematical modeling with modern scientific visualization techniques. A methodology has been developed that combines multiparameter modeling of dynamic processes with their real-time visualization. Using differential equations of motion and finite element methods, a mathematical model has been developed that takes into account the aerodynamic and mechanical characteristics of the assembly under study. The results of numerical modeling using modern CAE systems are presented, including stress-strain state and aerodynamic analysis. Particular attention is paid to the development of algorithms for visualizing and interpreting multidimensional data.

Keywords: digital twin, scientific visualization, mathematical modeling, wind turbine, numerical methods, CAE systems, multiparameter modeling.

1. Introduction

In today's world, where digitalization and intellectualization are becoming key factors of competitiveness in manufacturing industries, digital twin technology (Digital Twins are becoming increasingly important. The use of digital twins is particularly relevant during the development of complex and expensive technical systems, allowing for design optimization, risk reduction, and faster time-to-market for innovative products.

This article explores the use of digital twin technology to optimize the design of a wind turbine blade assembly currently under development. It presents the experience of a team from the Keldysh Institute of Applied Mathematics of the Russian Academy of Sciences in creating a digital twin designed for high-fidelity virtual testing and modeling.

This study, presented in a series of papers [1-7], makes a valuable contribution to the fields of mathematical modeling, numerical methods, and high-performance computing, enabling the creation of digital twins that accurately reflect the physical processes occurring in real systems. The developed technology enables parametric optimization of blade assembly design, taking into account various design constraints and maximizing wind turbine efficiency.

The aim of this paper is to demonstrate the capabilities of a digital twin, created by a team from the Keldysh Institute of Applied Mathematics (RAS), for optimizing the design of a wind turbine blade assembly during the development phase. The paper describes the workflow and reveals how the application of modern mathematical modeling and visualization methods enables the identification of optimal geometric parameters that ensure maximum wind turbine efficiency and mitigate the risks associated with implementing new technology. Using the Navier-Stokes equations, the open object-oriented OpenFOAM library, and modern computing resources, such as the K-100 hybrid computing cluster at the Keldysh Institute of Applied Mathematics (RAS), it is possible to create an accurate model simulating airflow around the blade assembly. Particular attention is paid to the visualization of parametric analysis results, which allows for the determination of the optimal blade assembly geometry, taking into account mass and moment of inertia constraints.

The article will present the results of numerical experiments demonstrating the effectiveness of the developed approach and the prospects for its application to the design of other complex technical systems.

2. Digital Twin: Concept and Possibilities

A digital twin is a virtual copy of a physical object or system that is constantly updated with data obtained from the real prototype. This allows not only to monitor the current state of the object but also to predict its behavior under various conditions and optimize its performance.

It's important to distinguish between a digital twin and a digital shadow. A digital shadow can predict the behavior of a real object only under the conditions under which the data was collected, but cannot simulate situations in which the real object was not used. A digital twin, on the other hand, allows for the simulation of a wide range of scenarios and virtual testing, which is especially important in developing new products and optimizing existing ones.

In the context of developing complex and expensive installations, such as wind turbines, creating a digital twin becomes a powerful tool for optimizing the design process and reducing the risks associated with implementing new technology. During the development phase, when a physical prototype does not yet exist or is in the early stages of creation, a digital twin enables virtual testing and simulation of various design solutions, identifying potential problems, and optimizing system parameters before costly physical production begins.

Wind turbines, even at the design stage, are complex systems that require consideration of numerous factors, including aerodynamics, material strength, load dynamics, and energy conversion efficiency. Traditional design methods, based on sequentially completing individual stages and physically testing prototypes, can be time-consuming, expensive, and do not always allow for the early identification of all potential problems.

A digital twin enables an iterative design process, in which virtual testing and simulation are conducted at each stage of development, enabling rapid evaluation of various design solutions and identification of optimal system parameters. Through the continuous exchange of data between the digital model and the results of physical experiments (if any), the digital twin becomes an increasingly accurate and reliable tool for predicting the behavior of the actual system. This reduces the risks associated with implementing new technology, reduces development time and costs, and improves the efficiency and reliability of the final product.

The economic feasibility of creating a digital twin during the development of wind turbines is driven by several factors. First, it reduces the costs of physical testing and prototyping through virtual experiments. Second, it reduces development time by accelerating decision-making and identifying potential issues early. Third, it improves the efficiency and reliability of the final product by optimizing system parameters based on virtual testing results. Fourth, it reduces the risks associated with implementing new technology through virtual testing and modeling various scenarios.

The concept of digital twins is actively developing in the Russian economy, as evidenced by works [8-11]. A separate paradigm for their application on a global economic scale has been developed – Smart Digital Twin [(Simulation & Optimization)- Based Smart Big Data Driven Advanced (Design & Manufacturing), the driver of which is a “smart” digital twin, formed as a result of multidisciplinary multi-scale numerical modeling and the application of many optimization technologies [8].

In the context of complex and expensive installations such as wind turbines, creating a digital twin is becoming not just desirable but practically essential to ensure their efficient and safe operation. Wind turbines operate under highly uncertain conditions, exposed to variable wind loads, temperature fluctuations, corrosion, and other factors that can lead to performance degradation and accidents. Traditional monitoring and diagnostic methods based on periodic inspections and historical data analysis are often insufficient for the timely detection and prevention of potential problems.

Creating a digital twin during the development of wind turbines is a strategically important step, allowing for the optimization of the design process, mitigation of risks, increased efficiency and reliability of the final product, and reduction of development time and costs. This technology opens new horizons for the development of wind energy and contributes to the creation of more efficient and reliable energy systems.

3. Creation of a digital twin of a wind turbine blade assembly

To effectively create a digital twin of a wind turbine blade assembly, it is necessary to combine the efforts of specialists from various fields: design engineers, technologists, materials scientists, calculators, and others. This leads to the emergence of a new type of engineer – a systems engineer – possessing competencies in various fields of knowledge and capable of effectively interacting with a variety of modeling and analysis tools.

The process of creating a digital twin involves several stages, each of which plays a vital role in ensuring the adequacy and accuracy of the virtual model. The key stages are listed below:

1. Construction of a CAD model.
2. Construction of the computational grid.
3. , the problem of flow simulation around the blade assembly is solved based on the Navier-Stokes equations using specialized software such as OpenFOAM [12-16].
4. Visualization of results.

The model developed for simulating the operation and variation of blade assembly shape is a unified process chain of algorithms. This chain includes the construction of a CAD model to describe the complex blade assembly geometry, the generation of a computational mesh based on the resulting geometry, the solution of a flow simulation problem based on the full system of Navier-Stokes equations, and the visualization and animation of the results in mono- and stereo modes. Based on this unified process chain of algorithms, technologies for varying blade assembly shape based on key parameters have been developed to determine the optimal blade assembly shape in terms of performance characteristics.

The objective of this study is to find the optimal shape for a power plant blade assembly in terms of power load. It is necessary to find a shape that provides maximum torque while varying three key geometric parameters of the assembly—two blade pitch angles and blade width. The potential increase in the assembly's mass and moment of inertia should not significantly exceed those of the base assembly shape.

The calculations were performed using the open-source object-oriented OpenFOAM library, written in C ++, which supports massive parallelization mechanisms and is intended for numerical modeling of continuum mechanics problems [12-14]. The library is based on finite-volume approximations written in operator form. OpenFOAM elements are actively used in industry, academia, and the expert community, in particular, in the numerical analysis of the energy characteristics of horizontal-type plants [14-16]. All calculations for solving the problems of mathematical modeling of the flow around a power plant were performed on the K-100 hybrid computing cluster at the Keldysh Institute of Problems of Materials Science, Russian Academy of Sciences [17] in parallel computing mode. The complete system of Navier-Stokes equations, describing the motion of a viscous, heat-conducting, compressible gas, was used as a mathematical model [18]. To analyze the results, a method for animated visual representation of the operation of the blade assembly in mono- and stereo-modes [3] was developed, based on modern methods and concepts of visual display of the results of numerical calculations.

As a result of previous work, a computer technology was developed that enables mathematical modeling of a power unit's blade assembly with specific geometric parameters and determines the force load acting on the unit in the airflow. This created the basis for further research to determine the optimal blade assembly shape for wind load, taking into account technological constraints.

Building a robust and effective numerical technology involves several necessary steps, including inputting geometric data, processing it, constructing a mesh model, selecting specialized flow calculation algorithms, analyzing current results, calculating functionals (forces and moments), visually representing the results, and processing and analyzing the results of multivariate calculations. The foundation of this technology is the process of mathematical modeling of a wind turbine blade assembly with specific geometric parameters under a given wind load. The mathematical modeling process itself involves creating an experimental computational system for calculating the flow around the complete wind turbine assembly based on solving the Navier-Stokes equations and calculating the corresponding integral flow characteristics.

The next important stage is the creation of a methodology for conducting optimization calculations to select the optimal shape of a wind turbine based on the optimization of the values of the selected integral characteristics.

The mesh generation process begins with the description of the surface bounding the three-dimensional body being considered. Typically, the surface of a real three-dimensional object with a complex shape is fully or partially imported from a CAD package and can be edited if necessary. Surface meshes have fairly "obvious" requirements—no self-intersections, closedness, and a few others. When working with real industrial objects, the resulting CAD surfaces rarely meet these criteria, and additional surface preparation is required to obtain a volumetric computational mesh of the required quality.

To achieve this goal, a number of automatic tools with a wide range of quality control tools are used, such as tools for creating and editing surface meshes as " surface wrapping " and " remeshing ", which allows to reduce the time spent by orders of magnitude and almost completely eliminate the need for manual mesh preparation. " Surface Wrapper corrects CAD geometry defects (closes holes, eliminates self-intersections, etc.), resulting in a closed surface with the desired level of detail. To create a high-quality initial triangular surface mesh, use the " surface " tool. remesher ", which allows, using a fairly wide range of settings, to obtain a surface with the required parameters (the degree of smoothness in areas with high curvature, the degree of resolution of thin areas, the rate of growth of the characteristic size of surface cells when moving away from areas with high detail, the preservation of topological features, local mesh refinement, etc.).

Fig. 1 already presents an “error-free mesh” – a triangular surface mesh with the necessary parameters (degree of smoothness in areas of high curvature, level of resolution of fine points, rate of growth of the characteristic size of surface cells with distance from areas of high detail, preservation of topological features, local mesh refinement, etc.).

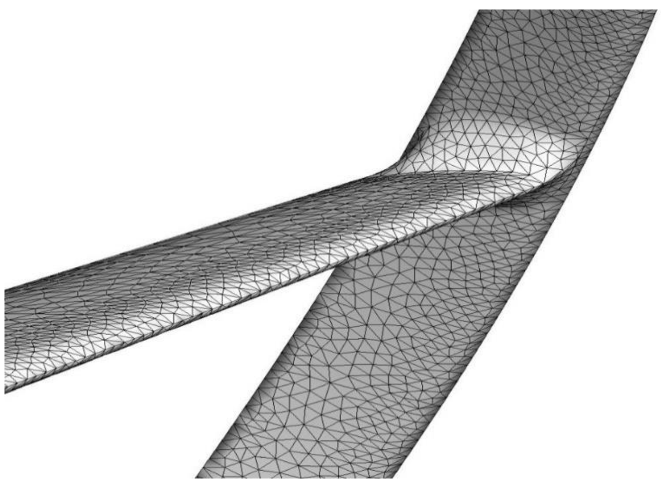

Fig. 1. Corrected surface triangular mesh constructed for the original CAD surface

The resulting surface mesh forms the basis for constructing a volumetric computational mesh. The specifics of numerical simulation of liquid and gas flows dictate certain rules for constructing volumetric meshes. When solving such problems, the volumetric mesh typically consists of two main parts: a prismatic mesh near the flowed surfaces and an arbitrary polyhedral mesh at a sufficient distance from the surfaces.

Currently, flows of a continuous gas medium (i.e., a gas medium under the assumption that it can be considered without taking into account individual particles) are calculated on the basis of the Navier-Stokes equations. This system of equations is as follows, see, for example, [18].

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

$$\frac{\partial (\rho \vec{u})}{\partial t} + \nabla \cdot (\rho \vec{u} \otimes \vec{u}) = -\nabla p + \nabla \cdot (\boldsymbol{\tau}_m + \boldsymbol{\tau}_t)$$

$$\frac{\partial (\rho E)}{\partial t} + \nabla \cdot (\rho \vec{u} H) = \nabla \cdot \left[\vec{u} \cdot (\boldsymbol{\tau}_m + \boldsymbol{\tau}_t) + (\vec{q}_m + \vec{q}_t) \right]$$

Here $\vec{u}$is the average flow velocity vector with components (u, v, w); τ_m и τ_i- molecular and turbulent (obtained by averaging various functionals from small-scale pulsations) components of the viscous stress tensor; $E = e + |\vec{u}|^2/2$ - specific total energy of the gas, e - specific internal energy of the gas $H = E + p/\rho$- total enthalpy; $\vec{q}_m$ и $\vec{q}_t$- molecular and turbulent components of the heat flux density vectors.

The type of the remaining turbulent components is no longer universal; their selection involves so -called turbulence models. Turbulence models must be selected taking into account the properties of real physical flows within the selected parameter range. Note that the influence of turbulence on the physical characteristics of the entire process was not considered in the calculations performed.

To determine the optimal wind turbine geometry for load bearing, a series of blade assembly models were constructed with varying blade geometry. We selected three key parameters, varying which allows us to describe a wide variety of possible geometric shapes.

Fig. 2 schematically shows these variable parameters: $\gamma_1$is the angle between the direction of the main blade and the vertical, $\gamma_2$is the angular size of the main blade in the direction of the axis of rotation, L is the width of the main blade. Note that throughout the following we will present the results for the three parameters (γ_1', γ_2, L), where $\gamma_1' = 90° - \gamma_1$.

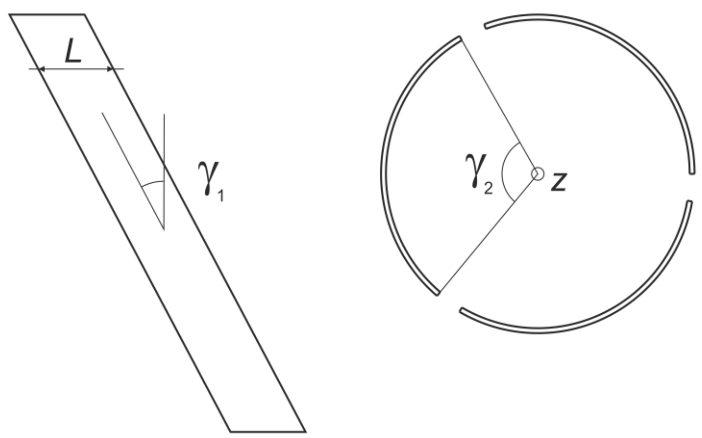

Fig. 2. Variable geometric parameters $(\gamma_1,\ \gamma_2, L)$ for determining the optimal shape of the blade assembly geometry.

To determine the main parameters and dimensions of the product (spacing between supports, average blade cross-sectional shape, etc.), the wind turbine geometry was used, obtained by scanning a prototype test sample. Surface construction and modification were performed using the SolidWorks CAD package. All constructed models were exported in SLDPRT format.

The values (γ_1', γ_2, L) change in a certain range, with the base value being $(\gamma_1')_{баз}$, $(\gamma_2)_{баз}$,$L_{баз}$ the result of laser scanning of the prototype was selected: $(\gamma_1')_{баз}$= 55 degrees, $(\gamma_2)_{баз}$=120 degrees, $L_{баз}$~ 20 cm. γ_1'takes on values of 45, 50, 55, 60 and 65 degrees, γ_2 — values of 100, 110, 120 and 130 degrees, L is selected from the set of values $0.8\ L_{баз}(L-20\%)$, $0.9L_{баз}(L-10\%)$, $L_{баз}$, $1.1\ L_{баз}\ (L+10\%)$,$1.2\ L_{баз}(L+20\%)$.

For clarity, let us supplement the schematic picture presented in Fig. 2 with three-dimensional images, see Figs. 3-4. The drawings represent the change in the shape of the blade assembly with variations in the geometric parameters described above.

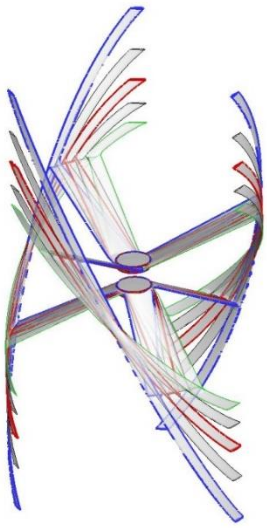

Fig. 3. Changing the angular parameter γ_1': 45 (green), 50 (gray), 55 (basic version, red), 60 (gray) and 65 (blue) degrees. $\gamma_2=(\gamma_2)_{баз}$, $L=L_{баз}$.

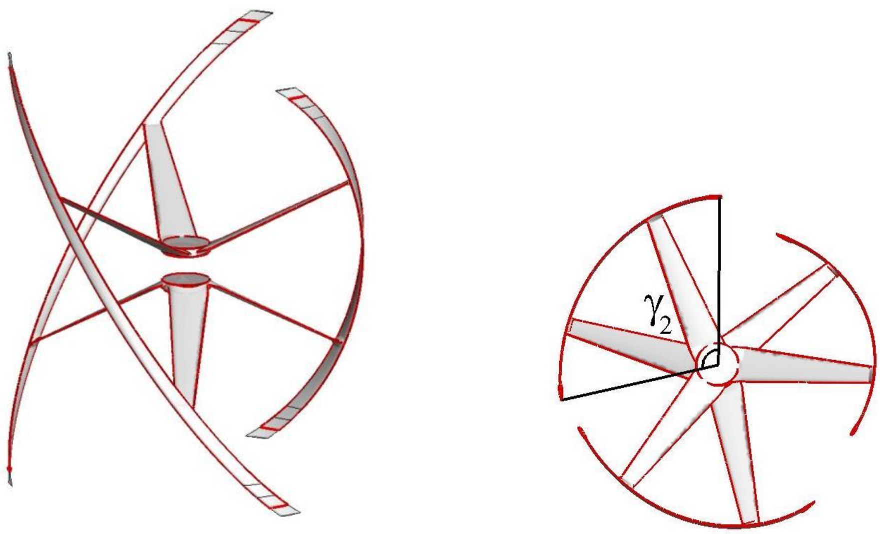

Fig. 4. Changing the angular parameter γ_2: 100, 110, 120 (basic version, red) and 130 degrees. In this case, the parameters $\gamma_1'=(\gamma_1')_{баз}$, $L=L_{баз}$.

The formal general statement of the optimization problem is as follows: find among the elements x that form the set X such an element x^* on which the given function $f(x)$ reaches a minimum (or maximum) value, i.e. $f(x^*) = \min_{x \in X} f(x)$ (or $f(x^*) = \max_{x \in X} f(x)$, respectively). Therefore, in order to formulate an optimization problem, it is necessary to specify: the feasible set $X = \{\vec{x} | g_i(\vec{x}) \leq 0, i = 1, \ldots, m\} \subset R^n$, the objective function $f(x): X \to R^n$, and the search criterion – what we are looking for – (max or min).

Solving such a problem means either finding the desired extremum $f(x^*) = \min_{x \in X} f(x)$ or showing that no solution exists. If, when specifying a feasible set of X constraints, $g_i(\vec{x})$ are absent, then we are dealing with an unconstrained optimization problem. If the conditions $g_i(\vec{x})$ exist, then such a problem is called a constrained optimization problem.

Assuming a number of design constraints inherent to the problem under consideration, we formally deal with a conditional multiparameter optimization problem, which is typical of design optimization problems in general. Our case is a multiparameter optimization problem, since variations of three key geometric parameters are considered. The variations of these parameters are limited by ranges and, therefore, impose constraints on the varied parameters. In general, the exploratory parametric search problem can be formulated as follows: find the values of the key geometric parameters of the blade assembly $x^* = ((\gamma_1')^*, \gamma_2^*, L^*)$ that ensure the maximum value of the objective function M* = max M(γ_1', γ_2, L). The main aerodynamic characteristics were chosen as the objective function : the total aerodynamic force $\vec{F}$ and torque $\vec{M}$.

To solve the optimization problem, we propose using a grid method, which is appropriate for an initial assessment, as multidimensional problems are significantly more complex and time-consuming than one-dimensional ones. The essence of the proposed method for finding the smallest value is to determine the values of the objective function at a discrete set of nodes that do not exceed the feasible set X. In other words, the ranges of variation for each key geometric parameter are partitioned at a specific step.

Thus, the spatial region defined by the parameter ranges is covered by a grid. The objective function is calculated at each grid node. The largest of the set of objective function values on a given grid is taken as the maximum. Previously, this method was traditionally considered practically unsuitable for problems of higher dimension due to the long calculation time required. However, the development of parallel computing allows for calculations to be accelerated by orders of magnitude. This makes the most unpretentious and simple methods truly applicable to practical problems. Moreover, their simplicity and reliability give them significant advantages in this context.

The conducted numerical simulations allowed for exploratory research and a rough optimization estimate of the optimal set of key geometric parameters for maximizing blade assembly torque. To achieve this, each key parameter γ_1', γ_2, L was sequentially varied, while the two remaining parameters were held constant.

An analysis of the calculated values for the blade assembly volume and its moment of inertia for various assembly geometry variants reveals that variations in angular parameters and blade width have different effects on the geometric characteristics—the assembly volume and moment of inertia. Specifically, width variations lead to a more significant increase in volume and moment of inertia, making it pointless to consider the constraints associated with combined variations in angular parameters and blade width. Therefore, a decision was made to consider design constraints for angular parameter variations separately for each data layer corresponding to a given blade width.

Below are the results of taking into account design constraints according to a previously developed methodology [7] for varying angular parameters γ_1' and γ_2 for a given blade width L+20%.

In Figure 5, the calculated volumes are represented as a three-dimensional surface $V_i(\gamma_1', \gamma_2)$ depending on the variations in the main angles. The plane bounded by red corresponds to the volume value for the base case. Accordingly, the plane bounded by blue corresponds to the volume value for the base case increased by 10%. The intersection lines of both planes with the surface of volume values limit the variation of the product shape. In Figure 6, the range of acceptable values, taking into account the volume constraints, is shown on the plane of angular parameter variations and is enclosed in the area bounded by the thick red and blue lines.

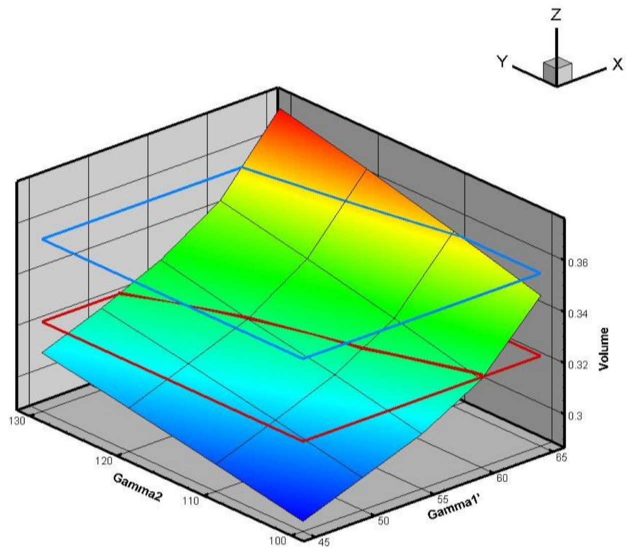

Fig. 5. Organization of accounting for volume restrictions for the L+20% data layer.

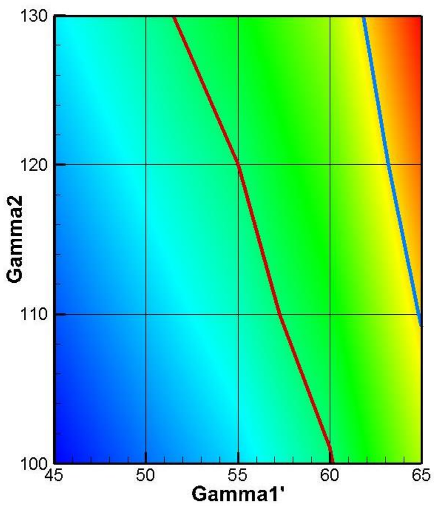

Fig. 6. Volume limitation area for varying angles for the L+20% data layer.

Limits for the moment of inertia are determined in a similar manner. They are shown in Figures 7 and 8. In Figure 7, the calculated moments of inertia are represented as a three-dimensional surface $M_i(\gamma_1', \gamma_2)$depending on the variations in the main angles. The plane bounded by red corresponds to the moment of inertia value for the base case. Accordingly, the plane bounded by blue corresponds to the moment of inertia value for the base case increased by 10%. The intersection lines of both planes with the surface of the moments of inertia values limit the variation in the product shape. In Figure 8, the range of permissible values is represented on the plane of angular parameter variations as the area between the thick red and blue lines.

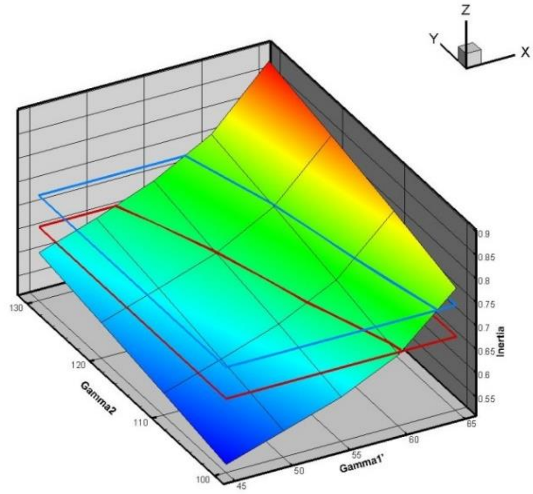

Fig. 7. Organization of accounting for restrictions on the moment of inertia for the L+20% data layer.

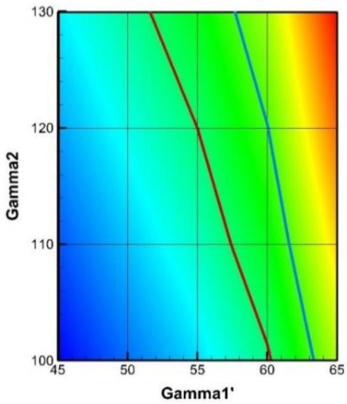

Fig. 8. Limitation region for the moment of inertia when varying angles for the L+20% data layer.

Thus, for a given data layer corresponding to a blade width of L+20%, we have obtained the volume and moment of inertia constraints shown in Figures 6 and 8. Now we need to organize their combined consideration. To do this, they should be combined in a single image and the range of angle variations selected that corresponds to the most stringent constraint.

For a more visual representation, let us consider Figure 9. This representation allows us to more accurately determine the range of variation of the angular parameters that ensures the maximization of the torque, taking into account the imposed restrictions on the change in volume and the moment of inertia of the blade assembly.

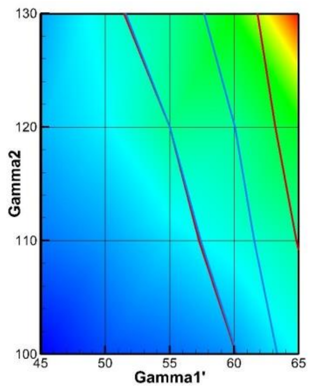

Fig. 9. Two-dimensional representation of the distribution of the moment of inertia taking into account the limitations when varying the angular parameters.

Figure 9 provides a clear picture of the desired range of angular parameter variation. It can be stated that, given the selected volume and moment of inertia constraints, the desired range lies within the angle variation range $\gamma_1'$of 55° to 60°, and the angle variation range $\gamma_2$of 120° to 125°.

4. Development Prospects: From a Specialized Solution to a Digital Engineering Paradigm

The digital twin technology developed and tested by a team from the Keldysh Institute of Applied Mathematics of the Russian Academy of Sciences using a wind turbine blade assembly as an example represents not only an effective solution for optimizing a specific design but also a potentially transformative paradigm in digital engineering. The achieved results demonstrate the feasibility of creating a scalable and adaptive platform capable of integrating various methods of mathematical modeling, numerical analysis, and data visualization to solve a wide range of problems in various industries.

Further development of the technology involves in-depth research and expansion of the platform's functionality in the following areas:

1. Developing adaptive modeling methods: Adaptive modeling methods need to be developed that automatically adjust the level of model detail depending on the problem being solved and available computing resources. This will optimize the modeling and analysis process, ensuring the required accuracy of results at minimal cost.
2. Research into methods for verifying and validating models: Rigorous methods for verifying and validating models, based on comparison of modeling results with experimental data and the results of other independent calculations, will increase confidence in the modeling results and ensure the reliability of decisions made.
3. Developing tools for automatic model generation: Tools for automatically generating models based on data about the geometry, materials, and operating conditions of a system offer potential for reducing the time and cost of creating digital twins and making them accessible to a wider range of users.

The developed technology can be applied to solve a wide range of problems in various industries, including:

- Design and optimization of complex technical systems: Developing digital twins for aircraft, automobiles, power plants, and other complex technical systems will optimize their design, improve efficiency and reliability, and reduce development and operating costs.
- Monitoring and diagnostics of technical condition: Developing digital twins for monitoring and diagnostics of equipment's technical condition will enable timely detection of defects and prevention of accidents, as well as optimization of maintenance and repair schedules.
- Complex Process Management: Developing digital twins for complex process management will optimize equipment operation, improve production efficiency, and reduce energy and material costs.
- Development of new materials and technologies: Developing digital twins to model the properties and behavior of new materials and technologies will accelerate their development and implementation into production.

Implementing these promising development areas will require the consolidated efforts of the scientific community, industrial enterprises, and government agencies, as well as significant investment in research and development. However, the results of this work will create a powerful platform for digital engineering, which will contribute to the competitiveness of Russian industry and strengthen Russia's position in the global high-tech market.

5. Conclusion

This paper presents the experience of a team from the Keldysh Institute of Applied Mathematics of the Russian Academy of Sciences in developing and applying digital twin technology to optimize the design of a wind turbine blade assembly. It is demonstrated that creating a digital twin enables highly accurate virtual testing and modeling, reducing development and testing time and costs, and enabling the identification of optimal geometric parameters for maximum turbine efficiency. The developed technology represents an effective solution, readily scalable and applicable to a wide range of engineering problems. It can be used for the design and optimization of complex technical systems in various industries, as well as for monitoring and diagnostics of their technical condition.

Prospects for further development of the technology include expanding the platform's functionality, developing specialized model libraries, integrating with computer-aided design systems, and applying machine learning and artificial intelligence methods. Implementing these promising areas will create a powerful digital engineering platform that will enhance the competitiveness of Russian industry.

In conclusion, digital twin technology is a key area of development in modern science and technology. Its application allows us to solve complex engineering problems, create innovative products, and improve production efficiency. Further research and development in this area will contribute to the development of the Russian economy and improve the quality of life.

Acknowledgments

The calculations were performed using the K100 hybrid supercomputer installed at the Keldysh Institute of Applied Mathematics and Mathematics Supercomputer Center of the Russian Academy of Sciences.

References

1. Bondarev A.E., Zhukov V.T., Manukovsky K.V., Novikova N.D., Feodoritova O.B. Development and organization of mathematical modeling of the flow around a stationary blade of a power plant. Preprints of the Keldysh Institute of Applied Mathematics of the Russian Academy of Sciences, 2014, No. 60, 19 p. URL : http : // library. keldysh. ru / preprint. asp ? id = 2014-60.
2. Andreev S.V., Bondarev A.E., Bondarenko A.V., Vizilter Yu.V., Galaktionov V.A., Gudkov A.V., Zheltov S.Yu., Zhukov V.T., Ilovaiskaya E.B., Knyaza V.A., Manukovsky K.V., Novikova N.D., Ososkov M.V., Silaev N.Zh., Feodoritova O.B., Bondareva N.A. Modeling and visualization of a complex-shaped power plant in stereo animation mode // Proceedings of the 25th International Conference on Computer Graphics and Vision GraphiCon'2015, Protvino, Russia, September 22-25, 2015, pp. 183-187.
3. Andreev S.V., Bondarev A.E., Bondarenko A.V., Vizilter Yu.V., Galaktionov A.V., Gudkov A.V., Zheltov S.Yu., Zhukov V.T., Ilovaiskaya E.B., Knyaza V.A., Manukovsky K.V., Novikova N.D., Ososkov M.V., Silaev N.Zh., Feodoritova O.B., Bondareva N.A. Modeling and visualization of the operation of a complex blade assembly in a power plant / Scientific visualization, v.7, N 4, pp.1-12, 2015. http :/ / sv - journal. org /2015-4/01. php ? lang = ru
4. Andreev S.V., Bondarev A.E., Bondarenko A.V., Zheltov S.Yu., Zhukov V.T., Ilovaiskaya E.B., Manukovsky K.V., Novikova N.D., Ososkov M.V., Feodoritova O.B. Modeling and visualization of the operation of a complex-shaped power plant / Mathematical modeling, 2016 (in press).
5. Galaktionov V.A., Bondarev A.E., Zhukov V.T., Feodoritova O.B., Novikova N.D., Manukovsky K.V., Andreev S.V., Mikhailova T.N., Ryzhova I.G. Development of a methodology for computer calculation of wind turbine (WT) blades and creation of a numerical model of a wind turbine // Report of the Keldysh Institute of Problems of Applied Mathematics of the Russian Academy of Sciences, No. 12/15, 2015, 58 p.

6. Galaktionov V.A., Bondarev A.E., Zhukov V.T., Feodoritova O.B., Novikova N.D., Manukovsky K.V., Andreev S.V., Mikhailova T.N., Ryzhova I.G. Solution of the optimization problem with the aim of finding the optimal shape of the product design from the point of view of the power load // Report of the Keldysh Institute of Problems of Materials Science RAS, No. 25/15, 2015, 49 p.

7. Galaktionov V.A., Bondarev A.E., Zhukov V.T., Feodoritova O.B., Novikova N.D., Manukovsky K.V., Andreev S.V., Mikhailova T.N., Ryzhova I.G. Implementation of a refined solution to the problem of determining the optimal shape of a blade assembly and determining the permissible range of changes in the geometric parameters of the assembly // Report of the IAM named after. Keldysh RAS, No. 2/16, 2016, 27 p.

8. Borovkov A. I., Ryabov Yu. A., Maruseva V. M. New paradigm of digital design and modeling of globally competitive products of the new generation//Digital production. Methods, ecosystems, technologies. Moscow: Department of Corporate Training, Moscow School of Management Skolkovo, - 2018. - P. 5, 24-43.

9. Borovkov A.I., Shcherbina L.A., Maruseva V.M., and Ryabov Yu.A. “The global technological agenda and global trends in industrial development in the context of the digital economy” Innovations, - 2018. - N 12 (242) - P.33-42.

10. Borovkov A.I., Maruseva V.M., Ryabov Yu.A. “Smart” digital twins — the basis of a new paradigm of digital design and modeling of globally competitive products of the next generation // Springboard to success. — 2018. — No. 13. — P.12–16. — URL: http://assets.fea.ru/uploads/fea/news/2018/04_april/12/tramplin-uspeha_13-16.pdf

11. Borovkov, A. I. Digital twins: definition, approaches, and development methods // Digital transformation of the economy and industry: Proceedings of the scientific and practical conference with foreign participation, St. Petersburg, June 20–22, 2019 / Edited by A. V. Babkin. — St. Petersburg: Federal State Autonomous Educational Institution of Higher Education “Peter the Great St. Petersburg Polytechnic University”, 2019. — P. 234–245. — DOI 10.18720/IEP/2019.3/25.

12. T. Maric, J. Hopken, K. Mooney. The OpenFOAM technology primer. www.sourceflux.de/book, 2014.

13. OpenFOAM http://www.openfoam.com

14. First Symposium on OpenFOAM ® in Wind Energy. 20 - 21 March 2013. http://www.forwind.de/sowe/Site/Home.html

15. Second Symposium on OpenFOAM ® in Wind Energy. May 19-21, 2014. http://wind.nrel.gov/2ndSOWE/

16. Third Symposium on OpenFOAM ® in Wind Energy. 15– 17 June 2015. https://www.eko.polimi.it/index.php/sowe2015/SOWE2015

17. Hybrid computing cluster K-100 URL: http://www.kiam.ru/MVS/resourses/k100.html

18. Landau L.D., Livshits E.M. Hydrodynamics. – Moscow, Nauka, 1986.