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POD-based Hydrodynamical Structures Visualization in Flows with an Internal Wave Attractor

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Abstract

Hydrodynamical structure attending a flow can be hid and hardly to reveal. One of the methods to find them is to use mode decomposition (such as Proper orthogonal decomposition, POD). The method represents the field given as a series of spatial modes multiplied by corresponding temporal coefficients. In the article the method is discussed applyingly to a complex flow with a wave attractor structure. Attractor modes present structured vortex-like figure which cannot be claimed to be aleatory.

As it turns out POD modes are not just a formal decomposition but have a physical origin: they are connected with instability cascade minor frequencies, as spectral investigation shows. Another consequence of that is that one of the collateral structure maximum can be visible. This proposition is proven as the structure is found to be visible in the flow itself.

Keywords: Wave attractor, instability, Proper orthogonal decomposition, visualization.

1 Introduction

It is known that there are systems where the ray-focusing phenomenon leads to the formation of specific structures called wave attractors. In such systems velocity magnitude (and kinetic energy) concentrates on a closed curve. This occurs because of specific dispersion relation (a wave conserves the angle with vertical axis rather than angle with the normal). Under such reflection law, ray focusing occurs just for geometrical reasons [1].

Since their discovery attractors were investigated in terms of main structure. But in recent work [2] vortex filaments in a system with inertial wave attractor were revealed due to vortex identification method. This experience makes us think that there can be other hid hydrodynamical structures.

For that purpose we will use another method than vortex identification: a mode decomposition. Being fed with a field, the method decomposes into characteristic modes, which can themselves be substructures of the flow.

Proper orthogonal decomposition [3] represents the solution as a series of spatial modes multiplied by corresponding temporal coefficients. The modes are selected so that they are eigenvectors of the solution's covariational matrix (solution matrix is a temporal slices ("snapshots") on the discrete spatial positions). Eigenvectors corresponding different eigenvalues are orthogonal (that's why the decomposition is called orthogonal). These modes can be used for dimension reduction, but we will be concentrated on the investigation of the modes obtained themselves.

2 The problem set

The problem of investigation came from the ocean dynamics. Comparatively recently the wave attractor phenomenon was discovered. It is a phenomenon of ray-focusing in liquid in the form of narrow curve where the fluid motion is concentrated, with the fluid remaining almost steady outside it. The conditions an attractor appears under are periodic external force, sloping side and salinity gradient [1].

Soon after the discovering the phenomenon was examined experimentally. As Leo Maas showed in [4], the trapezium with one sloping is an object good enough for the describing of ongoing processes. In the simplest case the wave generated reflects from each border one times, forming rhomboid-shaped internal wave, called (1,1) attractor, correspondingly the number of reflections. It's notable that coordinate of the rhomboid (reflection points) can be calculated analytically from external frequency and geometry. In [5] the method is introduced for (1,1) attractors; the recent work [6] generalizes it on (n,1) one. The numerical investigation turned out to be close to the experimental results [1][7], the later works [8][9] diverges less than in 10%.

In the pioneer works the external force was applied to the entire volume. Next, it was transferred to one of the sides [10] [11], which abled to provide a wider range of perturbations applied. The force was simulated via system called wave-maker that in the case consists of shafts and eccentrics providing discrete border perturbation.

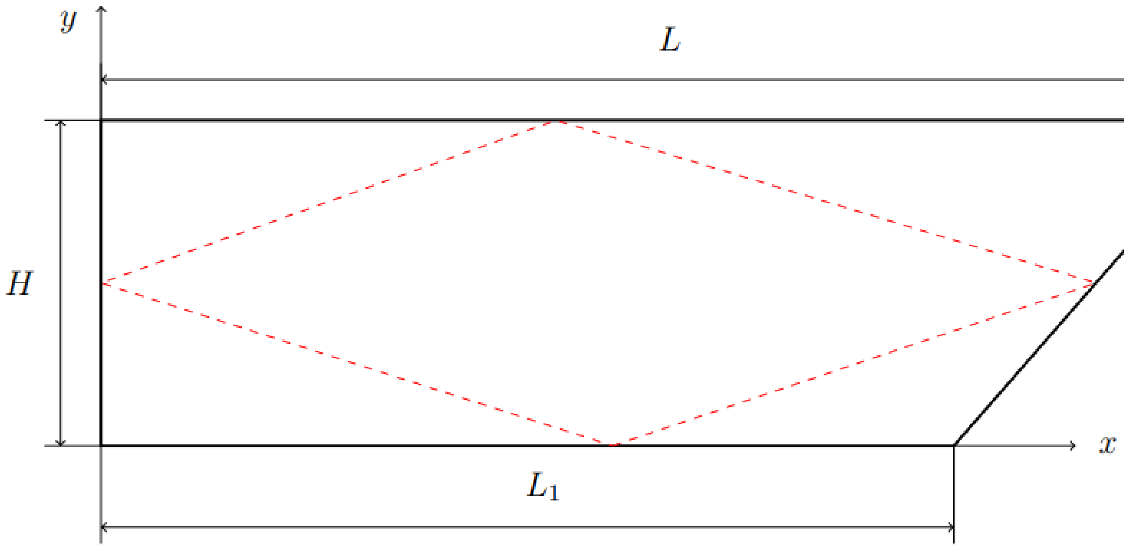

Figure 1: Scheme of domain geometry with attractor

That was the main setup and principles of a problem we base on. We will consider the model problem as two-dimensional one, hence it saves computational resources while providing enough accuracy [12][13][14].

Let's describe the setup to be simulated. The model region is a two-dimensional trapezium with one slope. Wave-maker is situated on the upper side, howbeit its position is not critical.

The equations system to solve consists of: the Navier-Stocks equation in Boussinesq approximation:

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}, \nabla)\vec{v} = -\frac{1}{\rho_m}\nabla\tilde{p} + \frac{\rho_s}{\rho_m}\vec{g} + \nu\Delta\vec{v} \tag{1}$$

and salt transport:

$$\frac{\partial \rho_s}{\partial t} + (\vec{v}, \nabla)\rho_s = \lambda_s \Delta\rho_s \tag{2}$$

$$\rho = \rho_m + \rho_s \tag{3}$$

Here $\tilde{p}$ is a pressure minus its hydrostatical part at ρ_m, ρ_m — density of fresh water, ρ_s — dissolved salt density, λ_s — salt diffusivity coefficient. Initial stratification was linear:

$$\rho_s|_{t=0} = \frac{H - y}{H}\rho_{s0}, \rho_{s0} = \text{const} \tag{4}$$

The system is supplemented by the incompressible continuity equation:

$$div\,\vec{v} = 0$$

The x-axis is supposed to be directed along the smaller (lower) trapezia base and y — along the vertical side wall. The scheme of the domain with the dimensions and schematic attractor curve is shown on Fig. 1.

The initial condition for velocity is zero:

$$\vec{v}|_{t=0} = \vec{0}$$

To emulate tidal forces a wave-maker was used. It perturbs one of the domain's side harmonically. Its position is of no matter, we placed it at the top of the domain.

Equation for border disturbance:

$$s(x) = a\sin(\omega_0 t)\sin(\pi x/L) \quad (5)$$

where $s(x)$ is an upper border profile, where $a$and ω_0 are the external forcing parameters; spatially the perturbance has a form of a half a sine. As soon as the perturbation is much smaller than the domain height we can rewrite the condition (5) as a velocity condition:

$$a\omega_0\cos(\omega_0 t)\sin(\pi x/L) \quad (6)$$

This allows us to solve the problem in a fixed area, which saves computational resources with a minimal lack of accuracy.

On the other borders there are Dirichlet conditions for velocity.

For the salt we have impermeability condition $\frac{\partial\rho}{\partial n} = 0$ on all the boundaries. That's why we smoothed initial salinity on upper and lower walls.

Sizes of basin are: height $H = 40$ cm, length $L = 60$ cm, bottom length $L_1 = 3/4L = 45$ cm. Hence external force is periodic, we will also consider time values in units of its period T_0 ($T_0 = 1/f_0 = 2\pi/\omega_0$).

The problem was solved numerically via spectral element method (Nek5000 [15]); postprocessing was made using *python3* codes.

The feature of this problem is that such phenomenon as wave attractor can appear. It comes out as a closed curve the fluid motion is concentrated on. Fig. 2 shows the characteristical spatial distribution of the velocity in non-turbulent regime at a = 0.02 cm. The attractor is clearly visible.

As to temporal evolution, the velocity has oscillation with amplitude slightly modulated, with the exit to the saturation (Fig. 3).

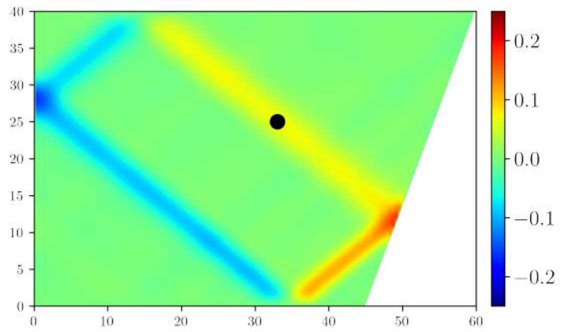

Figure 2: Vertical velocity component v_y distribution, a = 0.02 cm

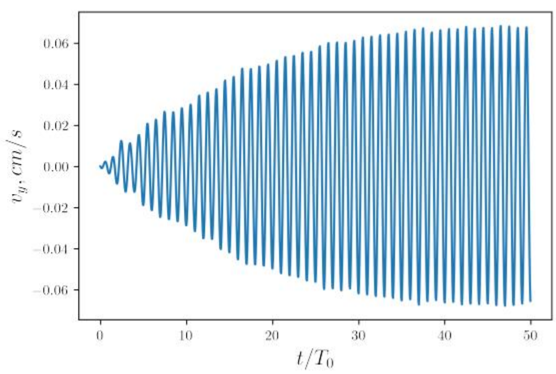

Figure 3: v_y evolution in the point mapped, a = 0.02 cm

3 POD application

For spatial mode extraction we applied Proper orthogonal decomposition [3]. The method decompose field into orthogonal modes timed by temporal coefficients:

$$u \approx \Sigma\Phi_i(x,y)T_i(t) \quad (7)$$

so that the covariance between modes tends to minimum. For numerical analysis we use intrinsic *python scikit* library code [16]. It turns out the streamlines of the modes will be more useful for representation rather than components, thus POD is made for both velocity component simultaneously.

POD decomposition seems to work suitable hence the spectrum of the flow is discrete, as Fig. 4 shows, which mean that the flow consists of a number of harmonics, and the decomposition 7 is physically justified. This occurs because of triadic resonance cascade in such system, whose presence in the system was previously proved [17][18][7] [19]. With such small external force amplitude at the harmonic on the externa force frequency f_0 dominates.

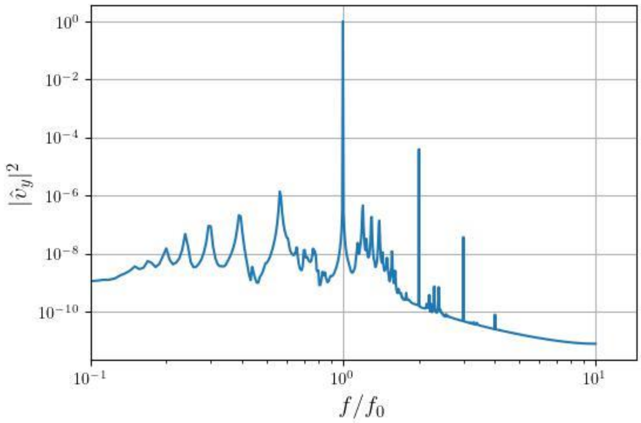

Figure 4: Energy spectrum of v_y

For the numerical investigation we used data on an interpolated uniform spatial grid (150x101 points) and 900 temporal snapshots.

Being applied to the solution, POD yields curious results. Alongside with the attractor mode (Fig. 5), which is supposed to be, there is several ordered vortex structures. The most powerful (in terms of energy share of the total one) one is those with four vortices oriented diagonally (Fig. 6). It's energy is notably high (39.5%), which makes it to be a regular and important structure rather than a relic.

Remaining modes are of some interest, and their existence could hardly be proposed. They presents two to four vortices (Fig. 7-9) located one over another ('vortex stack'). They have vanishingly small energy, but their ordered structure does not allow to consider them as an in-significant noise.

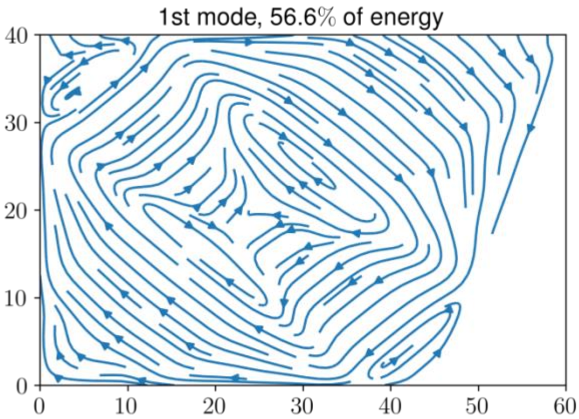

Figure 5: 1st eigenmode. Attractor structure

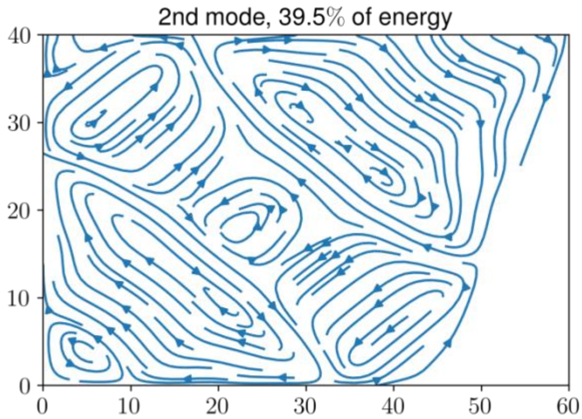

Figure 6: 2nd eigenmode. Quadrivorticular diagonal structure.

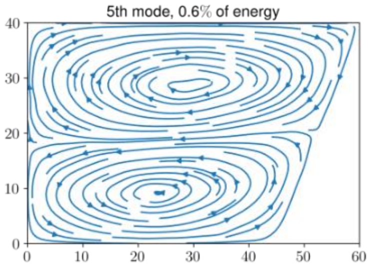

Figure 7: 5th eigenmode. Divorticular stack.

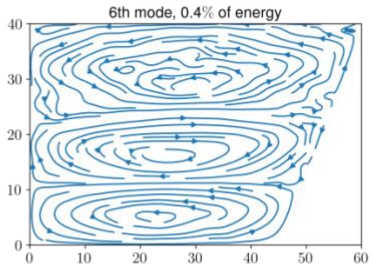

Figure 8: 6th eigenmode. Trivorticular stack.

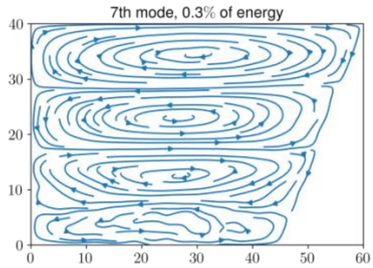

Figure 9: 7th eigenmode. Quadrivorticular stack.

With the spatial modes POD decomposition provides temporal coefficients associated (see 7). Fig. 10 shows those for modes 1st, 2nd and 5th modes (attractor, quadrivorticular structure and divorticular stack). If the first two modes are gazed, they turn out to be 'asheared' by a quarter a period, i.e., maxima of the 2nd coefficient take place where the first one meets zero. This phase delay can be proved by phase picture (Fig. 11). This allow to propose this mode to be visible in a pure flow (without decomposition), hence around the first mode coefficient's root the second will be the most powerful mode.

For a closer investigation spectra of the modes were made. Fig. 12 represents spectra for 1st, 5th, 6th and 7th modes (i.e., the main one and vortices stacks). The spectrum of the 2nd mode is very close to that of the 1st one and is deliberately not shown; their main difference is in phase (see above).

The spectra for the stack-like structures are discrete as well as that of the general flow, with the peaks being scattered along the frequency axis. This means that different modes corresponds different peaks of the spectra (as the modes have several peaks, they may partly overlap for different modes), which makes to think that the structures obtained (Fig. 7-9) are not accidental and result from the resonance cascade instability, hence this is a mechanism of spectrum saturation [20][17][18]. Our proposal is that these structures represent spatial resonances cascade, while the the spectral one is represented by collateral peaks on the spectrum (Fig. 4).

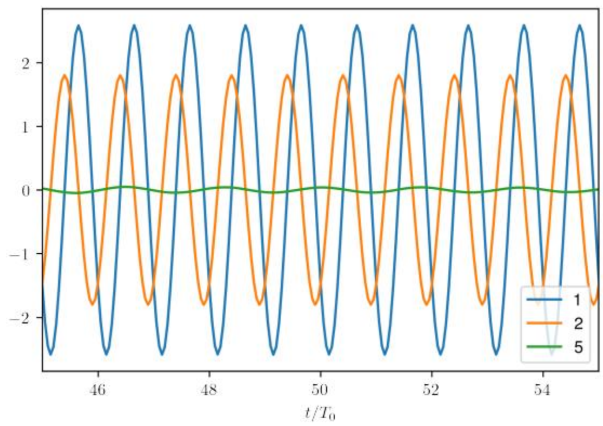

Figure 10: POD temporal coefficients

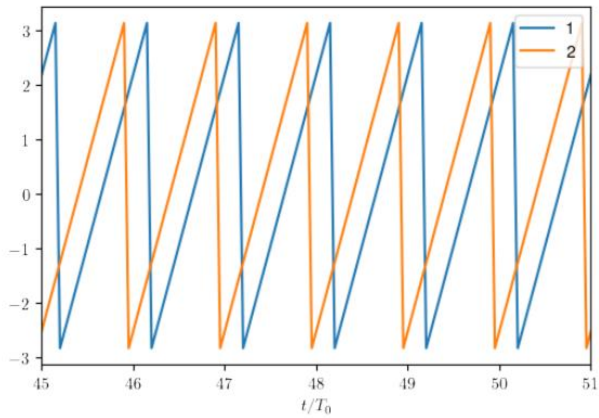

Figure 11: 1st, 2nd temporal coefficients phases

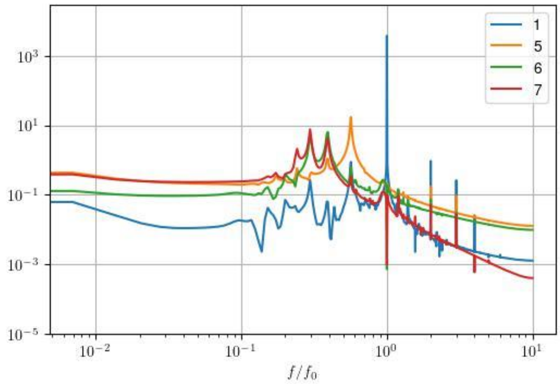

Figure 12: POD temporal coefficients spectra

4 Structure observation

As we noticed while discussing POD component temporal coefficients phases, the coefficient for the 1st mode have a delay correspondingly that of the 2nd one of some quarter of their common period, and hence the maxima of the second one occur when the 1st mode coefficient is near zero, we probably can see the structure like those represented at Fig. 6 in pure solution (without POD).

This proposal can be proved investigated flow streamlines picture evolution. Let's consider developed flow with attractor formed. The picture supposed to be seen is that shown on Fig. 13. Here t = 450 s or $45T_0$ which is enough for the ain structure establishment. Still, we observe four-vortex structure (Fig. 14) which is visible every $(n/2 + 0.35)\,T_0,\ n \in \mathbb{N}$. We emphasize that this is flow itself without any decomposition applied. This is how the hydrodynamical structures can be revealed after the prediction due to POD.

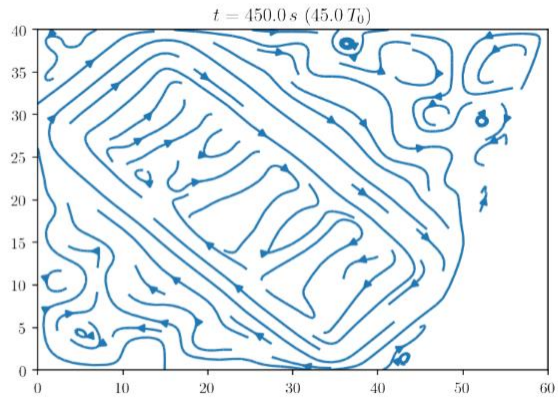

Figure 13: Streamlines of developed flow, $t = 450$ s $(45T_0)$. Main attractor structure

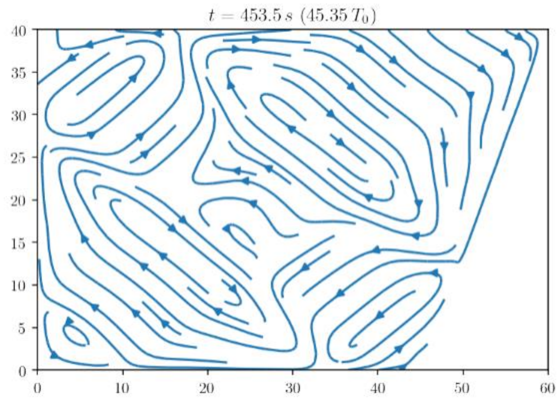

Figure 14: Streamlines of developed flow, $t = 453.5$ s $(45.35T_0)$. Quadrivorticular diagonal structure

Unfortunately, we cannot obtain the same result for the vortex stack-like structures. The reason is evident — their modes have frequencies different from those of 1sn and 2nd modes. Besides, their energies are very low to be directly visible, unlike that of the 2nd mode which is comparable with the energy of the attractor mode. These two points make their observation in the undecomposed flow nearly impossible.

5 Conclusions and Discussions

Revealing hydrodynamical structure in a flow, especially in a specific one, requires a very detailed investigation, hence the flow can be noised with a turbulence, or structures themselves may turn out to be not intense enough, which makes the search to be complicated. Decomposition tools, like POD, can help in revealing them. In the case of attractor problem we have decomposed the flow into vortex-like modes. Some of them turned out to be connected with instability minor frequencies. After a spectral investigation of POD temporal coefficients we managed to detect one of the structure (beyond the well-known rhomboid structure) found visible in the flow without decomposition. These aspects lead us to an important conclusion: POD modes are not just formal basis but structures of some physical sense. This allows to use the decomposition not only for dimension reduction, but for search of real physical structure attending the flow. Such possibility may be useful for ordered but low-intense structures that remain hid behind the main structure of a great energy. This makes POD to be a powerful instrument, especially in turbulent flows with instability vortex cascade, which on the general plane may seem only turbulent relics but can turn out to form ordered vorticular structures.

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