Scientific Visualization, 2022, volume 14, number 2, pages 49 - 61, DOI: 10.26583/sv.14.2.05

The Problem of Visualizing Solid Models as a Three-Parameter Point Set

E.V. Konopatskiy[1,A], A.A. Bezditnyi[2,B]

[A] Donbas National Academy of Civil Engineering and Architecture
[B] Sevastopol branch of «Plekhanov Russian University of Economics»

[1] ORCID: 0000-0003-4798-7458, e.v.konopatskiy@mail.ru
[2] ORCID: 0000-0003-0528-9731, bezdytniy@gmail.com

Abstract

This paper describes a new concept of defining geometric solids as a three-parameter set of points in three-dimensional space. An analytical description of such a three-parameter set of points is realized in the point calculus. The analysis of existing approaches to the visualization of a three-parameter set of points, which is described by a system of parametric equations, showed the absence of software solutions that are capable of visualizing three-dimensional solids in accordance with the proposed concept. In this connection the problem of visualization of solid-state models of geometrical objects parametrized in the point calculus is set. One solution to this problem is to adapt the existing way of visualization of three-dimensional objects in various simulation systems. Thus, on the basis of the point equation of a geometric solid, several surfaces are formed, the totality of which defines a closed envelope around the solid. Each of these surfaces corresponds to the limit state of one of the current parameters. Alternately fixing these parameters determines the point and parametric equations for their construction. Examples of visualization of face and curvilinear solids are given, which have shown the possibility of using computer algebra systems to visualize models of geometric solids in the form of a three-parameter set of points.

Keywords: solid modeling, geometric solid, point calculus, visualization, three-parameter point set, space dimensionality.

1. Introduction

Modern computer graphics industry, from automated, solid-state or information modeling systems (for example, BIM) to computer games, can no longer be imagined without solid-state modeling, which is often called three-dimensional in the domestic literature. In foreign sources, the most common term is “solid geometry” [1, 2]. Examples of the effective use of solid models can be works in mechanical engineering [3, 4], construction [5, 6], medicine [7, 9], education [2, 10], etc.

At the same time, the existing solid-state models are not inherently such, since they represent a hollow closed shell bounded on all sides by surfaces of different curvature. In accordance with [11], a solid in geometric modeling is a connected set of points located on the inner side of one outer and several inner shells located inside the outer shell, together with the points of these shells. In our opinion, this definition is too complicated for perception and requires clarification of additional terms. After analyzing the vector equations of solids given in [11], presented as a two-parameter set of points, we can conclude that these are equations of only the surfaces of the shell of a geometric solid. A similar concept is presented in other works related to the representation of geometric solids in various modeling and visualization systems [12-15].

In contrast, in [16, 17] a new concept of defining geometric solids is proposed, for which the existing approach is just a special case, and point equations of elementary geometric solids in point calculus (another name for BN-calculus) are obtained [18-20].

There is an opinion that solid models cannot be described by an equation. If we consider only the set of equations in an explicit form, then this is indeed the case and is connected with the fact that one of the axes of the coordinate system is reserved as a function. Accordingly, the number of variables of an explicit equation is always one less than the dimension of the space in which the geometric object is defined by this equation. At the same time, the point calculus, which uses the apparatus of projection onto the axes of the global coordinate system, makes it possible to use all coordinate axes, the system of which determines the space of the required dimension. This makes it possible to obtain the equations of geometric solids using simple arithmetic operations on the coordinates of points and continuous functions of parameters.

2. The concept of solid modeling of geometric objects in point calculus

Modeling of any geometric solids is inherently related to the dimension of the space in which the desired geometric solid is determined, and its topology. So, a line is a one-parameter set that can be defined at least in a 2-dimensional space. But at the same time, the line, both straight and curved, is itself a one-dimensional space. If you select a segment or an arc on a line, then, in fact, we get a one-dimensional analogue of a geometric solid, which is determined in point terms by the movement of the current point using the current parameter. The current point with its movement along a given trajectory fills the space, forming a continuous geometric object. Thus, a 1-dimensional geometric solid is a 1-dimensional space filled with points.

If we select an area on the surface that is bounded by lines and filled inside with points, then we get a 2-dimensional geometric solid that can be defined in 3-dimensional space, but is itself 2-dimensional. Moreover, such a 2-dimensional geometric solid, as well as the space in which it is located, is a two-parameter set of points and is determined by two current parameters. Accordingly, the geometric solid familiar to us in 3-dimensional space, as well as the space itself, is a three-parameter set of points and is determined by three current parameters. Thus, the three-parameter set of points in 3-dimensional space will be a geometric solid, and in 4-dimensional space it will be a three-parameter hypersurface. This approach has a generalization to a multidimensional space. For example, it can be used to describe the space-time continuum in 4-dimensional space.

Based on the above-mentioned considerations about the dimension of space, in [16, 17] another definition of a geometric solid was proposed. So, we will call a geometric solid a geometric set of points, in which the number of current parameters that determine it is equal to the dimension of space. The geometric solids defined in accordance with this concept can also exist in spaces of higher dimensions. This approach was effectively used for geometric modeling of multifactorial processes and phenomena by the method of multidimensional interpolation [21].

The definition of geometric solids in point calculus, by analogy with the geometric modeling of curves and surfaces, begins with the development of a geometric scheme. In this case, fixed points of the simplex (as a rule, they determine the overall dimensions of a geometric object), current points (they will fill the space with their movement, forming a geometric object) and intermediate points (needed for the convenience of compiling geometric and computational algorithms, but are subsequently excluded from the calculation) are distinguished. The process of interaction of these points is implemented using the moving simplex method [18], which is an analogue of the kinematic method of shaping geometric objects in point calculus. The geometric scheme is developed in such a way that the desired geometric object has predetermined geometric properties. The analytical description, first of all, depends on the geometric scheme, which predetermines the sequence of points definition. The final point equation is subjected to a coordinate-wise calculation, which is an operation of transition from point equations to a system of parametric equations of the same type. Thus, a computational algorithm is formed for computer simulation of a geometric object.

In our opinion, the proposed concept allows expanding the existing tools for modeling geometric solids. For example, its implementation makes it possible to find a new approach to solving the problem of synthesizing geometric solids from projection images [22–24].

I would like to note that within the framework of the proposed concept, point calculus is not the only possible mathematical tool for modeling geometric solids. Given that all point equations for their direct visualization are reduced to a system of parametric equations, they can be fully used to determine geometric solids, but this increases the number of calculations. In addition, images obtained in [25] in Figs. 12 with examples of constructing the form of thermal expansion in the form of discretely represented layers indicate the presence of the potential of the functional-voxel modeling apparatus for determining geometric solids in accordance with the concept proposed in [16, 17].

3. The problem of visualizing solid models

The problem of visualizing solid models comes from the concept of their definition. In the concept implemented at the moment in modeling systems, it is sufficient to model only the surface of the solid shell, without filling it with points. The technology of modeling and visual representation of surfaces is well studied and therefore does not pose any big problems in visualizing a geometric solid in the form of a closed set of surfaces (two-parameter sets of points). But if we proceed from the proposed concept of defining geometric solids in the form of a three-parameter set of points belonging to 3-dimensional space, then the question arises - how to visualize it? After all, the very concept of defining geometric solids is innovative and has not yet been implemented in any of the visualization systems for three-dimensional geometric models [26-28].

If you use the existing interpreters of computer mathematics systems, some of which have powerful tools for visualizing research results, then a similar problem arises. All of them can only visualize surfaces in 3D space, and their syntax does not allow visualization of a three-parameter set of points. Some of them contain their own set of simple geometric solids. For example, in the Plottools package of the Maple computer algebra system [29], there is a small library that allows you to visualize elementary geometric solids that can be built from the coordinates of vectors. But without the equations of these solids, it is practically impossible to work constructively with them.

Since the new concept of defining geometric solids involves filling a three-dimensional closed space with points, it is logical to use a discrete set of high-density points to visualize models of geometric solids. Computational experiments were carried out in the Maple software package.

As an example, consider a solid model of a triangular pyramid (Fig. 1), which, in accordance with [16], is determined by the following point equation:

$$M_1 = Auv\bar{w} + B\bar{v}\bar{w} + C\bar{u}v\bar{w} + Dw.$$

$$\Downarrow$$

$$\begin{cases} x_{M_1} = x_A uv\bar{w} + x_B \bar{v}\bar{w} + x_C \bar{u}v\bar{w} + x_D w \\ y_{M_1} = y_A uv\bar{w} + y_B \bar{v}\bar{w} + y_C \bar{u}v\bar{w} + y_D w, \\ z_{M_1} = z_A uv\bar{w} + z_B \bar{v}\bar{w} + z_C \bar{u}v\bar{w} + z_D w \end{cases} \quad (1)$$

where $\bar{u} = 1 - u$; $\bar{v} = 1 - v$; $\bar{w} = 1 - w$;
u, v, w are the current parameters that vary from 0 to 1;
A, B, C, D are any four points that do not lie in the same plane.

As a result, it was found that even with a small number of 10,000 points, there are difficulties in using such models due to the large computational load. Fig. 1 shows an example of visualization of a solid model of a pyramid in the form of a discrete set of points in the Maple software package. To visualize the triangular pyramid in the plan, 20 by 20 points were selected along the sides of the triangle, along the height of the pyramid – 30 points. The result is 12,000 points. At the same time, it is very difficult to work with the model on a 4-core computer with a frequency of 3.5 GHz, 16 GB of DDR4 RAM and a discrete GTX 1050 video card with 4 GB of video memory. Even the rotation of such a pyramid causes certain difficulties and the program freezes for a few seconds. And the necessary density of points in order to consider the solid as a single continuous object, as can be seen from Fig. 1 has not been reached.

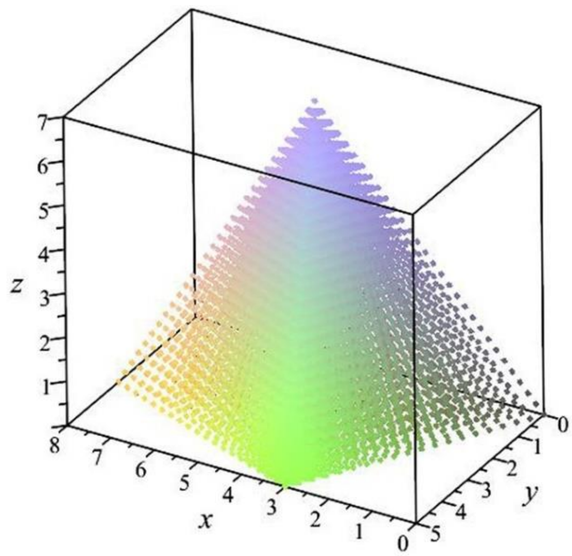


Fig. 1. Visualization of a solid model of a pyramid by a discrete set of points

Based on the experience of visualizing geometric solids in existing modeling systems, we will use the following method. We single out special cases from the point equation of a geometric solid, fixing in turn the necessary values of the parameters of the point equation. As a result, we obtain several partial surfaces, which together form a shell closed around the solid, by analogy with existing solid modeling systems. The visualization of such surfaces is well mastered and does not carry a significant computational load.

4. Visualization of faceted solids based on point equations

Let us consider several examples of visualization of faceted geometric solids, for which point equations have been obtained. In particular, in order to visualize the geometric model of a triangular pyramid, it is enough to select 4 planes that limit its surface. These planes are determined by the limiting values of the current parameters of the point equation (1). So, with the value of the parameter $w=0$, we get the plane of the base of the pyramid, and with $w=1$ - its top. In relation to visualization, the top of the pyramid, being a single point, does not carry much value, so it can be excluded from the visualization process. Moreover, it is duplicated by the side planes of the pyramids.

The side planes of the pyramid are determined by alternately fixing the parameters $u=0$, $u=1$and $v=1$. For a fixed value of the parameter $v=0$, we obtain a straight line of intersection of two side planes of the pyramid, which is a feature of the proposed parametrization in accordance with the geometric scheme given in [16]. This line can also be excluded from the visualization process due to dubbing. As a result, we get a visualization of a solid model of a triangular pyramid (Fig. 2).

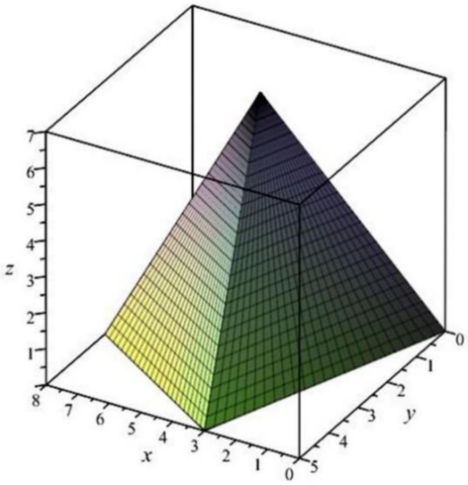


Fig. 2. Visualization of a solid model of a pyramid in the form of a set of 4 planes

Based on considerations of automating the visualization process, such exceptions cannot be made. It is enough to fix alternately all possible limit values of the current parameters and display the resulting planes on the screen.

Using the dot equation (1), one can easily obtain a truncated pyramid. To do this, it is enough to accept the parameter w value from 0 to 1. Thus, instead of a point, a fifth plane will be added. For example, when $w = 0,5$, we get the model presented in Fig. 3.

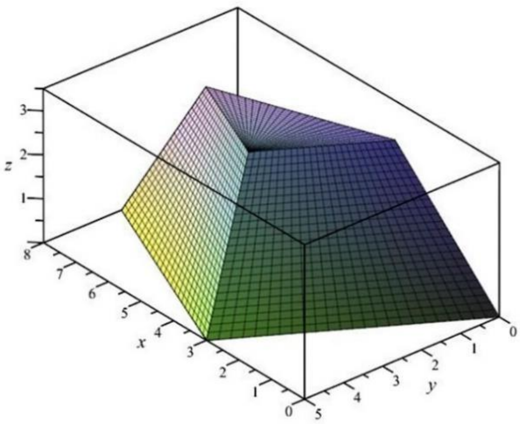


Fig. 3. Visualization of a solid model of a truncated pyramid in the form of a set of 5 planes

In this case, the sectional plane of the pyramid was parallel to the base. Consider an example of a section of the solid of a pyramid by a plane in general position, which we define using traces of the section plane on the sides of the pyramid $ABCD$:

$$\begin{cases} P = (A - D)p + D \\ Q = (B - D)q + D, \\ R = (C - D)r + D \end{cases}$$

where p, q, r are fixed parameters defining points P, Q, R (Fig. 4). Within the triangular pyramid, the values of these parameters should be taken on the interval from 0 to 1.

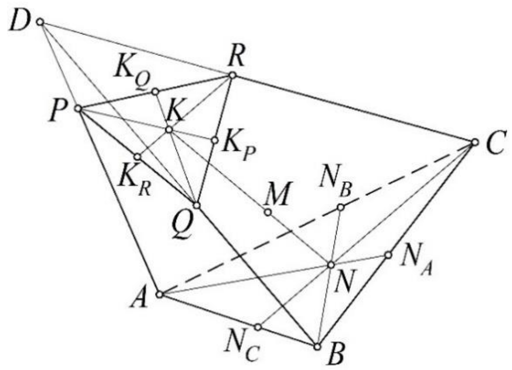


Fig. 4. Geometric scheme for modeling the solid of a pyramid truncated by a plane of general position

In accordance with [16], the equation for the base of a pyramid filled with dots has the following form:

$$N=(A-C)uv+(B-C)\bar{v}+C.$$

Similarly, the secant plane of the pyramid is determined, filled with points:

$$K=(A-D)puv+(B-D)q\bar{v}+(C-D)rv\bar{u}+D.$$

The space between these two planes will be filled with a movable segment NK:

$$M=N\bar{w}+Kw=(A-D)uv(\bar{w}+pw)+(B-D)\bar{v}(\bar{w}+qw)+(C-D)v\bar{u}(\bar{w}+rw)+D.$$

Having the final point equation of a geometric solid, to determine it in the global Cartesian coordinate system, it is necessary to perform a coordinate calculation by analogy with the equation (1):

$$\begin{cases} x_M=(x_A-x_D)uv(\bar{w}+pw)+(x_B-x_D)\bar{v}(\bar{w}+qw)+(x_C-x_D)v\bar{u}(\bar{w}+rw)+x_D \\ y_M=(y_A-y_D)uv(\bar{w}+pw)+(y_B-y_D)\bar{v}(\bar{w}+qw)+(y_C-y_D)v\bar{u}(\bar{w}+rw)+y_D \\ z_M=(z_A-z_D)uv(\bar{w}+pw)+(z_B-z_D)\bar{v}(\bar{w}+qw)+(z_C-z_D)v\bar{u}(\bar{w}+rw)+z_D \end{cases}.$$

As a result, we obtain a solid model of a pyramid truncated by a plane of general position PQR (Fig. 5).

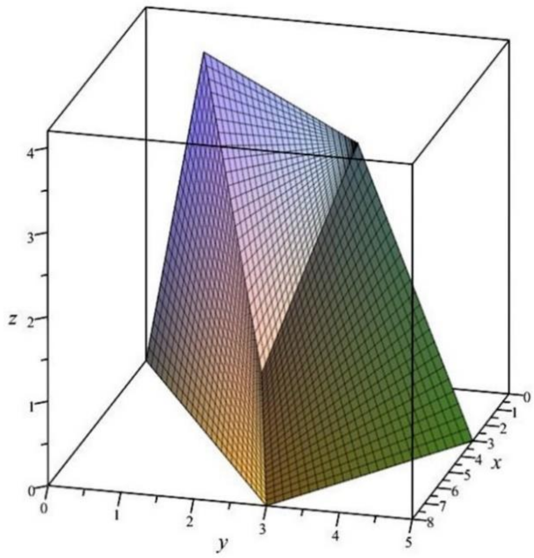


Fig. 5. Visualization of a solid model of a pyramid by a truncated plane in general position

In this case, to visualize the solid model, the limiting values of the current parameters are also used, which determine the five planes that bound the surface of the pyramid solid, which are determined from the general equation of the solid with the parameter values: $u=0$, $u=1$, $v=1$, $w=0$ и $w=1$.

As a result of the research, an interesting feature was discovered. Let us define in the simplex $ABCD$ the point equation of the truncated part of the pyramid, which is also a pyramidal solid with a secant plane PQR as the base:

$$M_2 = (A-D)\,pwuv + (B-D)\,qw\bar{v} + (C-D)\,rwv\bar{u} + D.$$

$$\Downarrow$$

$$\begin{cases} x_{M_2} = (x_A - x_D)\,pwuv + (x_B - x_D)\,qw\bar{v} + (x_C - x_D)\,rwv\bar{u} + x_D \\ y_{M_2} = (y_A - y_D)\,pwuv + (y_B - y_D)\,qw\bar{v} + (y_C - y_D)\,rwv\bar{u} + y_D \\ z_{M_2} = (z_A - z_D)\,pwuv + (z_B - z_D)\,qw\bar{v} + (z_C - z_D)\,rwv\bar{u} + z_D \end{cases}.$$

From the point of view of solid modeling, the truncated pyramid is the result of subtracting the solid of the truncated part $PQRD$ from the base one $ABCD$. At the same time, if we analyze the receipt of the equation, we get that the point equation of the truncated pyramid $ABCPQR$ is the sum of the truncated pyramid $PQRD$ and the base one $ABCD$: $M = M_1 + M_2$. Computational experiments on the visualization of geometric solids confirm this. Of course, the conditions for the interaction of geometric solids in point calculus still require additional research. But the result obtained already allows us to take a fresh look at the implementation of Boolean operations on solids, some of which can be replaced by simple arithmetic operations on points and their coordinates.

5. Visualization of curved solids based on point equations

A similar visualization method can be applied to curvilinear solids. As an example, consider the visualization of channel solids in the form of a surface with a wall thickness δ.

To parameterize the channel surface in point terms, a geometric algorithm is proposed (Fig. 6), which includes: determining the tangent NP to the guide curve ANB using parallel translation [18]; definition of the normal NN_1 using the metric operator of three points (it is an analogue of the scalar product of vectors in point calculus) and a point R fixing the length of the segment $|NR|$; definition of a binormal NN_2 by determining the exit point from the plane (it is an analogue of the vector product of vectors in point calculus) and a point Q that fixes the length of the segment $|NQ|$.

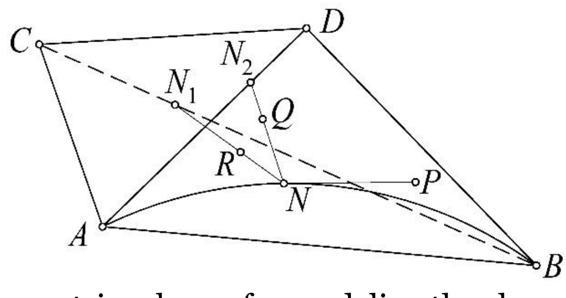


Fig. 6. Geometric scheme for modeling the channel surface

To pass from a channel surface to a channel solid, it is necessary to supplement the geometric model of the surface with a wall of constant or variable thickness. It is convenient to set such a wall using the conditional center N of the section of the channel surface (Fig. 7).

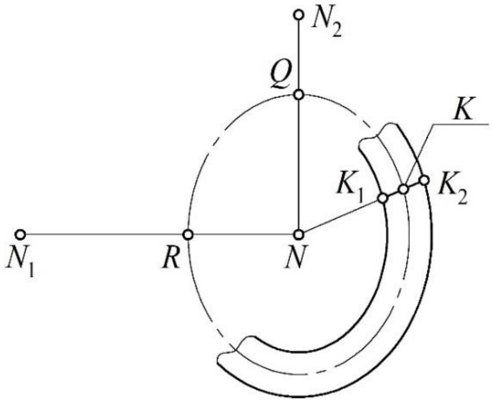


Fig. 7. Geometric scheme for determining the wall of the channel surface

Regardless of the shape of the generatrix of the region, it is determined by a moving point K, which fills the space with its movement, forming a closed curve. Any combination of continuous and compound lines can be used as closed curves. In this case, in fact, there is a rotation of the point K around the conditional center N. Select at the NK straight line segment K_1K_2 of length δ. The movement of this segment around the point N will ensure that the space is filled with points, forming a wall of the channel solid with a thickness of δ. We determine the point K_1 from the condition of belonging to a straight line NK:

$$\frac{KK_1}{KN}=\frac{|KK_1|}{|KN|}\Rightarrow\frac{K-K_1}{K-N}=\frac{\delta}{2|KN|}\Rightarrow K_1=(N-K)\frac{\delta}{2|KN|}+K,$$

where $|KN|=\sqrt{(x_K-x_N)^2+(y_K-y_N)^2+(z_K-z_N)^2}$.

We determine the point K_2 from the condition that the segment K_1K_2 is divided by the point K in half:

$$K_2=2K-K_1=(K-N)\frac{\delta}{2|KN|}+K.$$

Then the current point of the channel solid M will be determined by the following equation:

$$M=(K_2-K_1)w+K_1=(N-K)\frac{\delta(1-2w)}{2|KN|}+K\Rightarrow\begin{cases}x_M=(x_N-x_K)\dfrac{\delta(1-2w)}{2|KN|}+x_K\\ y_M=(y_N-y_K)\dfrac{\delta(1-2w)}{2|KN|}+y_K\\ z_M=(z_N-z_K)\dfrac{\delta(1-2w)}{2|KN|}+z_K\end{cases}.$$

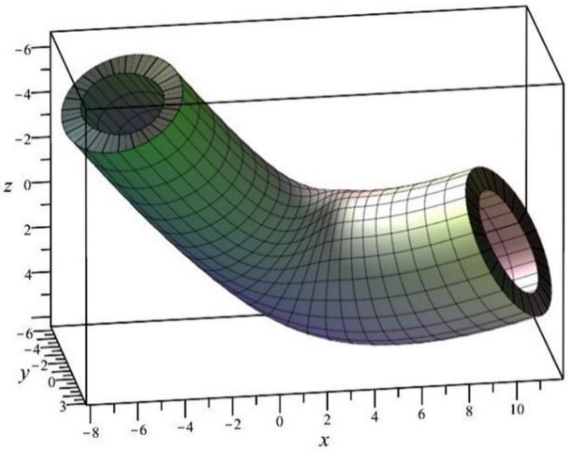
In a similar way, the wall thickness can be laid in or out of the generatrix of the channel surface.

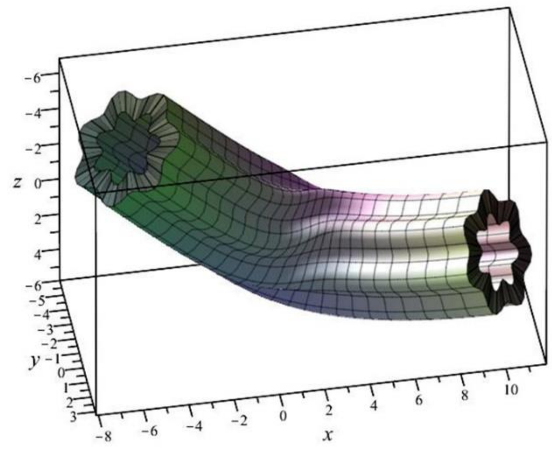
Next, we consider several examples of modeling channel solids in the form of a set of surfaces (Fig. 8a-8c), which are determined by successively fixing the following parameter values: $u=0$, $u=1$, $w=0$ и $w=1$.

In these examples of visualization of channel solids, a spatial transcendental line with a dotted equation is used as a guide:

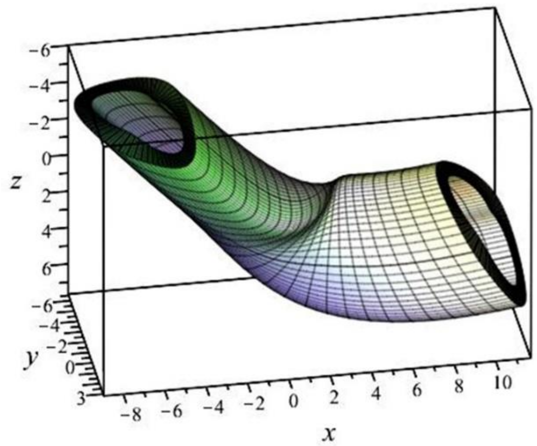
$$N=(A-D)u^3+(B-D)\bar{u}^3+(C-D)\sin(u)+D.$$

But the given geometric algorithm for shaping channel solids allows the use of flat curves along with spatial ones. Also, in addition to transcendental curves, any continuous or compound curves can be used.





a) b)



c)

Fig. 8. Visualization of the channel solid: a) with an elliptical generator; b) with a generator in the form of a sine wave [30] with 7 waves; c) with a generator in the form of a closed contour of the first order of smoothness [31]

6. Conclusion

As can be seen from the examples given, the proposed method is quite suitable for visualization in computer algebra systems of solid-state models in the form of a three-parameter set of points. At the same time, in our opinion, the problem of visualization of solid-state models in the form of a three-parameter set of points is urgent and requires additional research. Perhaps it is time to solve this problem radically, abandoning the use of monitors in favor of generating full-fledged three-dimensional images in space by analogy with holographic ones [32-34], since the proposed concept of solid modeling implies this.

It may seem that the implementation of the visual representation of the proposed concept of solid modeling is no different from the existing one, since it also uses a set of surfaces to visualize geometric solids. But the fundamental difference from existing systems is that in the process of modeling, instead of a set of several equations of surfaces, there will be only one-point equation in the computer's RAM. It will instantly give the special cases necessary for displaying on the computer screen in the form of separate surfaces bounding a geometric solid. At the same time, their point equations will be used to visualize the interaction of several geometric solids in the calculation process. In addition, the use of coordinate-based calculation of point calculus opens up the possibility of automatic parallelization of calculations of all coordinates of points to determine any continuous and discrete geometric objects. Therefore, the development of the proposed concept of solid-state modeling using point equations and computational algorithms based on them can be an effective tool for building computer graphics systems and solid-state modeling of a new generation. At the same time, the authors of the article will be pleased with new ideas on visualization of geometric solids based on point equations and joint cooperation in the development and implementation of highly efficient systems of geometric and computer modeling.

It should be noted that the proposed approach to the construction of geometric solids in the form of point equations can be an effective discretization tool for mathematical and computer modeling in finite element analysis systems. But when using the method of numerical solution of differential equations proposed in [35, 36] using geometric interpolants, this is not necessary, since a multidimensional geometric interpolant is a special superelement [37, 38] that carries information not only about geometric, but also about physical properties. Thus, solid-state modeling of geometric objects in point-to-point calculation together with numerical solution of differential equations using geometric interpolants is the theoretical basis for the development of a closed cycle of digital products, which, by analogy with the isogeometric method [39, 40] eliminates the need for coordination of geometric information in the process of interaction between CAD and FEA systems.

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