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Automating Placement of Point Feature Labels on a Digital Map using Non-linear Mathematical Programming

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Abstract

A paper considers one of the approaches to solving the problem of automating placement of point feature labels on a digital map. The causes of label overlaps are analyzed, and requirements for label placement are formulated. Two different statements of optimization problems are proposed for finding the optimal locations of labels of point feature labels on a digital map. One of them is based on the idea that as the distance between labels increases, the chance of labels overlay decreases, and as a result, it becomes possible to identify each point feature on a digital map and read information about it presented on the label. In another statement, it is proposed to minimize the distances from objects to their labels in order to quickly and efficiently identify a particular object on the map. The article presents the results of solving optimization problems, which show the fundamental possibility of obtaining optimal label locations and demonstrates the quality of label placement using the considered approaches.

Keywords: digital map, label, optimization methods, optimization problem, labels overlay.

1. Introduction

Digital maps are part of geographic information systems. The basis of construction of digital maps are topographic maps of the area. Digital maps are often used to move objects on the ground (or in air, water space).

Objects displayed on digital maps are, for example, ships and aircraft, road, and rail transport. All listed objects refer to point features: objects whose position in space is given by a single point [1]. On the map, such objects are displayed using conventional symbols. A conventional symbol is a graphic representation of an object that characterizes the displayed object in a certain way.

When displaying point objects on a digital map, several characteristics of the object and the symbol are considered. Each object has coordinates and a label - brief textual information describing the main parameters of the object. The label has dimensions and is tied to the coordinates of the object. Symbols of objects also have dimensions, that is, the maximum width and height, and are also tied to the coordinates of the objects.

To evaluate the mutual actions of objects, it is necessary to display a set of objects on a digital map at once. To identify a specific object on the map, along with the symbol of the object, its label is displayed. In various situations, labels can overlap each other, as well as cover part of the conventional signs. This situation leads to a decrease in the readability of the map, complicates the location of a particular object on the map, and makes it difficult to assess the interactions of objects.

The main factors in the occurrence of the object labels overlay:

- small scale of digital map;
- the presence of localization centers of objects in the area of the considered space.

At a small scale (the case of observing a large area of a digital map), a greater number of objects fall into the region of space reflected on the map compared to a large-scale map. As a result, the likelihood of object labels overlapping each other increases. Simply zooming in on the map to solve the problem of overlapping labels is not always effective. It may be the task of analyzing the situation on a large section of the map.

When analyzing the display of point objects on the map, it is possible to identify some features of their location in space. Such a feature is the presence of areas, let's call them the centers of localization of objects, where a significantly larger number of objects is observed in comparison with the rest of the space. When displaying aircraft on the map, airports, cities, etc. can be localization centers. Obviously, in such areas, the overlap of object labels is possible even at sufficiently large scales of the map.

In this paper, one of the methods for placing point feature labels on a digital map is considered: an automatic method, when an algorithm is used that calculates a certain location of labels in real time.

2. Automating placement of point feature labels

When placing object labels automatically, it is necessary to ensure:

- maximum readability of labels;
- the ability to unambiguously match a label with its object;
- sufficient speed when solving the problem of placement.

The first two requirements are quite clear. However, the criteria for «readability» and «unambiguous comparison» should be clearly defined since these concepts are quite different for each individual person. Providing the last requirement to allow using the results of solving the placement problem for moving objects, that is, for objects that quickly change their position on a digital map.

Currently, in many software tools that use digital maps, the problem of automatic placement of labels is solved in the most trivial way: the object label is displayed in one strictly fixed position relative to the symbol of the object. When there are many objects, some of the labels are not displayed at all, to avoid overlapping them. This approach can be observed, for example, on sites with a digital map to display aircraft in real time [2, 3]. Obviously, in this case it is impossible to uniquely identify a particular object.

One of the approaches to solving the problem of automatic placement of labels is the use of mathematical programming methods. So, in general case, among the set of possible solutions (in our case, the solution is a specific set of coordinates by which the labels are placed on the map), the solution is chosen that can be qualified as optimal in one sense or another according to some chosen criterion.

To apply the methods of mathematical programming, one should set an optimization problem by choosing a quality indicator (a functional, the extremum of which is sought using the optimization method) and imposing restrictions on the variables of the optimization problem (a set of label coordinates) [4].

We will analyze the initial data of the task and determine the quality indicator based on the requirements for placing the labels indicated above.

3. Initial data and agreements

Initial data in the task are the coordinates of object (x, y) in the coordinate system of the map (a specific pixel on the map). The map is made up of pixels, its width is L and its height is H pixels. The object itself is represented by a material point, the dimensions of the symbols are not taken into account.

In this paper, we will consider a standard rectangular label with a transparent base. For clarity, let the label display only the call sign of the object (for example, the aircraft callsign):

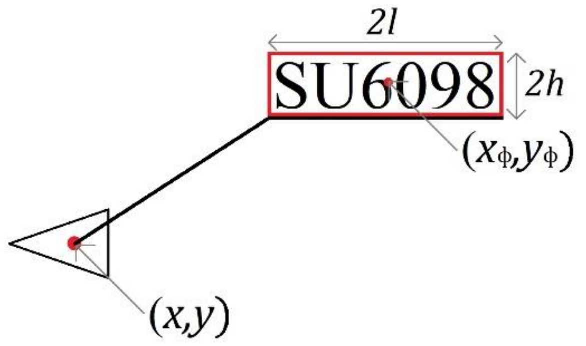

Fig. 1 Object (indicated by a triangle) and its label.

When setting the optimization problem, we will identify the label with a rectangle (marked in red in Figure 1). The object is located at the point with coordinates (x, y). The anchor point of the label is the center of the rectangle with coordinates $(x_{Ф}, y_{Ф})$. The letters l and h denote half the length and height of the label in map pixels, respectively (the length of the entire label is $2l$, the height is $2h$). For a unit of length, «halves» of the dimensions of the label are taken in order to avoid the appearance of unnecessary numerical coefficients in the formulas in the future. When solving the optimization problem, we will accept the constraint that for all objects the size of their labels is the same and equals $2l$ in length and $2h$ in height.

Another very important constraint is related to the numerical type of label coordinates. The label coordinates are integer (they determine the position of a pixel on a digital map). However, when setting the problem, it is permissible not to impose a restriction on the integer value of the coordinates of the labels. Such a «relaxation» significantly expands the set of approaches and methods that can be used to solve the problem. If any method for solving the set non-integer problem will provide maximum performance (in comparison with methods for solving integer problems), an error of ±0.5 pixels is quite acceptable. The obtained coordinates can be rounded up to integers and placing the labels at these coordinates will not impair the readability of the information from the labels.

Let's try to formulate the statement of the problem of automatic placement of labels in general. It is necessary to obtain coordinates of labels of objects $\{(x_{Ф_1}, y_{Ф_1}), (x_{Ф_2}, y_{Ф_2}), \dots, (x_{Ф_n}, y_{Ф_n})\}$, by the given coordinates of n objects, the size of the label and the size of the map. It is necessary that the location of the labels at these coordinates ensures the maximum readability of information from them for the unambiguous identification of a particular object on the map when placing labels on a digital map.

To formalize the subjective requirements of «readability» of information from labels and «unambiguous identification», we formalize the concepts of imposing a label on an object and imposing labels on each other.

Overlaying a label on an object means that the object enters the label area (rectangle). In other words, the label overlaps the object when the point with coordinates (x, y) falls within the region of the rectangle centered at the point $(x_{\Phi,} y_{\Phi})$.

We will divide the options for overlaying labels on top of each other into two groups: full overlays and partial overlays. Full overlap will be considered the situation when the center of one label (with coordinates $(x_{\Phi,} y_{\Phi})$) falls into the area of another label (rectangle). We will consider such a situation unacceptable, and the information contained in the labels unreadable.

We will designate the variant as partial overlap, when the centers of the labels do not fall into each other's areas, but the overlap area of the labels is not equal to 0. We will consider such a situation acceptable.

4. Statement of optimization problem

As mentioned above, the optimization problem is the problem of finding the extremum of the objective function (quality index) with a set of restrictions on the optimization variables. When it comes to the placement of labels, the optimization variables will be the coordinates of the object labels. Now it is necessary to decide which function will act as an indicator of the quality of placement of labels.

As an indicator of quality, you can consider the overlap area of the labels. With this approach, there is a need to calculate the area of overlap of one label on another. In the case when several labels overlap each other, and the number of labels is quite large, such calculations can greatly affect the performance (one of the requirements for automatic placement of labels given in Section 2). Therefore, other options for the quality indicator are proposed.

Let it be necessary to ensure the minimum overlap of labels on each other. This means that the greater the mutual distance between the labels, the higher the chance that the overlap will be minimal, and, as a result, the readability of information from the labels will increase. In this case, as a quality indicator, you can choose the total distance between the labels and solve the optimization problem as a problem of maximizing the total distance.

It is possible to formulate the problem with a different quality indicator. Let it be necessary to ensure the possibility of unambiguous identification of an object using its label. Then, the closer the label is to its object, the higher the chance of accurately matching the label with its object. As a quality indicator, we choose the total distance from objects to their labels. Then it is possible to set the optimization problem as the problem of minimizing the total distances.

Based on the above, we formulate the optimization problem in two versions: to the maximum and to the minimum.

5. Maximization of the distance between labels

The objective function for the distance maximization problem:

$$\sum_{j=1}^{n} \sum_{i=1}^{j-1} (\left(x_{\Phi_i} - x_{\Phi_j}\right)^2 + \left(y_{\Phi_i} - y_{\Phi_j}\right)^2) \to max, \tag{1}$$

where n is the number of labels, x_{Φ_i}, y_{Φ_i} and x_{Φ_j}, y_{Φ_j} are the coordinates of the centers of i and j labels.

Thus, we maximize the sum of squared distances between labels.

Let's put restrictions:

1. Restrictions on the distance between the object and the label:

$$(x_{\Phi_i} - x_i)^2 + (y_{\Phi_i} - y_i)^2 < K * 4(l^2 + h^2), \tag{2}$$

where l, h are the half-width and half-height of the label, respectively.

The distance from the object to its label should not exceed K squares of the label diagonal, where K is chosen empirically and then $K = 2$ is assumed.

2. Restrictions on the intersection of the label with the object:

$$\begin{cases} l^2 - \left(x_{\Phi_i} - x_j\right)^2 < 0; \\ h^2 - \left(y_{\Phi_i} - y_j\right)^2 < 0, \end{cases} \text{for } i = \overline{1,n}, j = \overline{1,n}. \quad (3)$$

3. Restrictions on overlapping labels:

$$\begin{cases} k_i^2 l^2 - \left(x_{\Phi_i} - x_{\Phi_j}\right)^2 < 0; \\ k_i^2 h^2 - \left(y_{\Phi_i} - y_{\Phi_j}\right)^2 < 0, \end{cases} \quad (4)$$

$$\text{for } i = \overline{1,n}, i = \overline{1,n},\ i < j,\ k_i \in [1,2],\ k_i \in R.$$

The coefficient k_i is set for each object. It defines the boundaries of the area around the i-label, which is not allowed to fall into the anchor point of the j-label. The minimum value of this coefficient is $k_i = 1$. The object, for which the coefficient is equal to one, has the lowest priority, and the maximum partial overlap is allowed for it (anchor point of the j-form (red) is on the border of the i-label (black), hit to the region of the i-label is limited):

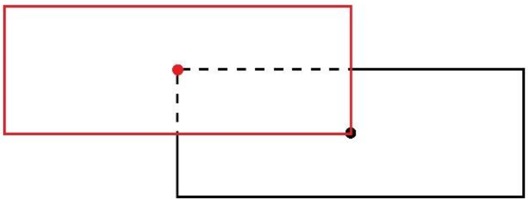

Fig. 2 Partial overlay ($k_i = 1$)

For $k_i \in (1,2)$, the anchor point of the j-label is not allowed to fall into a rectangular area of size ${k_i}^2(l * h)$ (yellow in Figure 3):

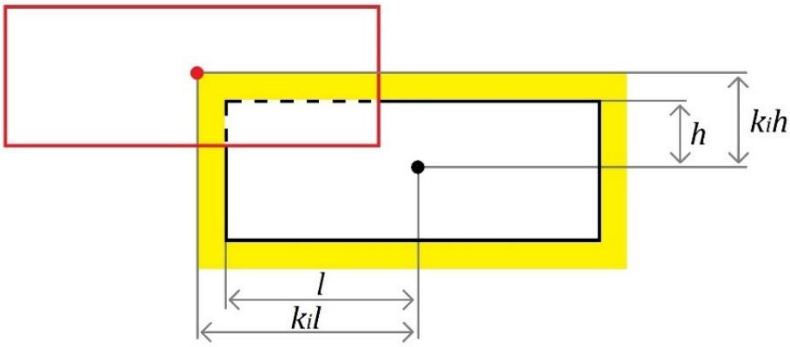

Fig. 3 Partial overlay ($k_i \in (1,2)$)

The maximum value that the coefficient k_i can take is 2. An object for which the coefficient is equal to 2 has the highest priority, and partial overlap is not allowed for it (the anchor point of the j-label (red) does not fall into a rectangular area of size $4l * 4h$ (yellow on figure 4)):

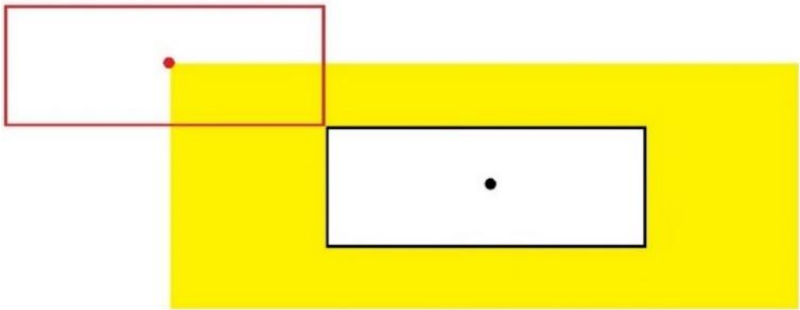

Fig. 4 No overlay ($k_i = 2$)

4. Restrictions on going beyond the boundaries of the map:

$$\begin{cases} x_{\Phi_i} - (L - l) < 0; \\ y_{\Phi_i} - (H - h) < 0; \\ l - x_{\Phi_i} < 0; \\ h - y_{\Phi_i} < 0, \end{cases} \text{for } i = \overline{1,n}. \quad (5)$$

Here L and H are the width and height of the map in pixels.

The total number of optimization problem constraints is $3n^2 + 4n$.

6. Minimization of the distance between labels and objects

Such an objective function is proposed to minimize the distance between labels and objects:

$$\sum_{i=1}^{n}((x_{\Phi_i} - x_i)^2 + (y_{\Phi_i} - y_i)^2) \to min, \quad (6)$$

where n is the number of labels, x_i, y_i are coordinates of the i-object, x_{Φ_i}, y_{Φ_i} are coordinates of centers of i-label.

We minimize the sum of squared distances between objects and their labels.

Restrictions:

1. Restrictions on overlapping labels:

$$\begin{cases} \left(x_{\Phi_i} - x_{\Phi_j}\right)^2 - k_i^2 l^2 > 0; \\ \left(y_{\Phi_i} - y_{\Phi_j}\right)^2 - k_i^2 h^2 > 0, \end{cases} \quad (7)$$

for $i = \overline{1,n}, i = \overline{1,n}$, $i < j$, $k_i \in [1,2]$, $k_i \in R$.

2. Restrictions on the intersection of the label with the object:

$$\begin{cases} (x_{\Phi_i} - x_j)^2 - l^2 > 0; \\ (y_{\Phi_i} - y_j)^2 - h^2 > 0, \end{cases} \text{for } i = \overline{1,n}, j = \overline{1,n}. \quad (8)$$

3. Restrictions on going beyond the borders of the map:

$$\begin{cases} (L - l) - x_{\Phi_i} > 0; \\ (H - h) - y_{\Phi_i} > 0; \\ x_{\Phi_i} - l > 0; \\ y_{\Phi_i} - h > 0, \end{cases} \text{for } i = \overline{1,n}. \quad (9)$$

The total number of optimization problem constraints is $3n^2 + 3n$.

7. Solution of the optimization problem

To solve the set optimization problems, the authors developed an application written in Python. The optimization of the objective function was carried out using the interior point algorithm for high-dimensional non-linear programming problems. This algorithm is implemented in the SciPy library for scientific computing [5]. Figure 5 shows solutions to the problem of maximizing the distance between labels and minimizing the distance between labels and objects, respectively.

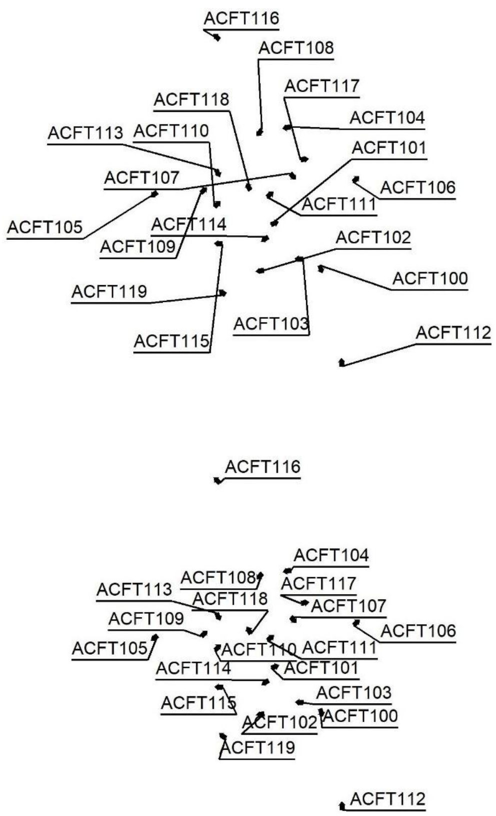

Fig. 5 Solutions of problems of maximizing the distance between labels (top) and minimizing the distance between labels and objects (bottom) for L = 1920, H = 1080, l = 90, h = 28, $K = 2$, $k_i = 2$ for $i = \overline{1,20}$

The obtained solutions satisfy the requirement of «readability» of information from the labels, each object is uniquely identified. However, it should be noted that the formulated optimization problems do not exclude the intersection of extension lines (lines connecting the object and its label) of labels, which can also affect the quality of information perception. Refusal to display extension lines, of course, reduces the image «entanglement», but this approach is acceptable only in a situation where the objects are located relative to each other at a distance greater than half the diagonal of the label. With a greater object accuracy, it becomes impossible to unambiguously compare each of them with its own label.

In the obtained optimal solutions of the minimization problem (1–5), there are significantly fewer intersections of extension lines than in another optimization problem (6–9). Obviously, the total length of extension lines is greater when solving the maximization problem (6–9), which increases the probability of their mutual intersection (Fig. 5). Thus, it can be argued that it is more expedient to solve the minimization problem in order to ensure better «readability».

Separately, it should be noted that the solution of the minimization problem requires less calculations. When comparing the objective functions (1) and (6), it is clear that to calculate the second one, it is required to perform fewer addition and exponentiation operations. Also, there are fewer restrictions in the formulation of the minimization problem, which also affects the performance. This assumption is confirmed experimentally. On average, the minimization problem is solved faster by 4–6%. In the results shown (Fig. 5), the optimal solution to the problem of maximizing the distance between labels was obtained in 6.53 s, and the solution to the problem of minimizing the distance between labels and objects was obtained in 6.19 s. So, solving the minimization problem is also more profitable from the point of view of performance.

In general, the dependence of the time for solving optimization problems on their dimension (the number of objects) has a quadratic character. Modification of the proposed problem statements to exclude the intersection of extension lines will inevitably lead to the complication of the system of restrictions. The number of restrictions will increase by n^2, since it will be necessary to check all extension lines in pairs for intersections. Based on the total number of restrictions in the problems already presented, when trying to eliminate the intersections of extension lines, one should expect an increase in the time to find the optimal solution up to 30%.

To apply the approach given in this article to solve the problem of automatic placement of labels in the practical implementation of a GIS, it is recommended to divide the initial data (a set of point objects) into subgroups that include a small number of objects (10–15 pieces). And further optimization problems should be solved in parallel for each subgroup. This approach can potentially provide the performance required in each system.

8. Conclusion

This article describes an approach for solving the problem of automatic placement of labels of point objects on a digital map using methods of nonlinear mathematical programming. Two options for setting optimization problems are proposed: with a quality index in the form of a total distance between labels and with a quality index in the form of a total distance between objects and their labels. When solving optimization problems, restrictions on labels overlapping each other, restrictions on labels overlapping objects, as well as restrictions on labels leaving the map are considered. The fundamental possibility of solving the problems posed with the help of the interior points algorithm is shown, and the obtained optimal solutions are given.

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