Approach to conditioning improvement in scalar calibration problem for three-axis accelerometer module using visualization of measurement efficiency function

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Abstract

The paper proposes a way to optimize a set of calibration angular orientations of accelerometer module using efficiency function visualization for stationary based calibration to increase an accuracy of estimated error model parameters. It includes an analysis of mathematical model of three-axis accelerometer module measurement errors. The method improves an estimation accuracy of analyzed error model parameters. The error model includes following factors: angular errors in sensor alignment in module frame of reference, deviations of sensor scalar coefficients and sensor biases. Measurement efficiency function characterize an impact of each newly made measurement on overall problem conditioning and depends on module angular position relative to calibration reference vector. By determining minima points of shown function, it is possible to form an optimal set of angular positions for calibrated module, which allows to achieve better conditioning of calibration problem. These minimal points are determined via optimization algorithm. Due to function complexity the visualization is necessary to find and set initial points for searching. The approach is verified by computer simulation which shows that optimal set of angular module positions (optimized set), formed by presented method, improves estimation accuracy of considered parameters in error model in presence of errors in angular positioning of module during calibration process, in comparison with non-optimized set.

Keywords: scalar calibration, accelerometer module, function visualization, least-square, Kalman filtering.

1. Introduction

One of the key factors of INS accuracy is calibration of an accelerometer module which is the essential part of navigation system. Calibration is an estimation process performed to determine parameters of the error model that describes how sensor errors form. One of the traditional approaches to calibrate an accelerometer module is a fixed base calibration, done by measuring a g-vector, perceived in module frame of reference in different angular orientations of a turn table. The discrepancy between expected and actual measurements can be used to estimate parameters of error model that caused these errors [1, 2, 3, 6]. This problem can be described as an inconsistent system of linear equations and can be solved via least-square methods or Kalman filtering. A set of orientation angles chosen for calibration determines a conditioning of a problem and therefore robustness of the estimation result to unaccounted disturbances in measurements. Typically, the main source of such disturbances are errors in angular positioning of turn table [1, 4]. In practice orientation angles are usually chosen to guarantee full observability of estimated parameters without taking into consideration a conditioning of an obtained system [3, 5, 6, 7]. By choosing optimal orientation set for calibrated module it is possible to get a well conditioned system of

equation and therefore increase calibration accuracy. The paper introduces an efficiency function of particular measurement which is convenient to use for determining proper module alignments by finding its extreme points via optimization algorithm. In approach shown in [8] some of angular orientations are found numerically to maximize determinant of observation matrix in case of strictly orthogonal sensor alignment for vector calibration method, but no validation with virtual modeling or hardware are shown. This paper proposes a method to form a set of optimized orientation angles for best system conditioning on every calibration measurement with no necessity in orthogonal sensor alignment inside a module. In order to find extreme points of the efficiency function with numerical optimization algorithm this function must be visualized.

The paper contains an approach to form such efficiency function for each of the calibration measurements. The parameters of error model are then estimated to fit the discrepancy between measurements. The main feature of scalar calibration is a linear system that establish linear relation between vector of model parameter deviation values and discrepancy between absolute expected value of perceived acceleration and its measured absolute value [7]. Presented method allows to chose various equally optimal orientations for each measurement due to periodic property of an efficiency function which may be useful in case of asymmetry in scaling coefficients.

2. Mathematical model and calibration method.

The model includes 3 accelerometers mounted on gyro-stabilized platform so that their axes E_1, E_2 and E_3 are orthogonal to each other. Nominal alignment of second accelerometer axis E_2 is co-directional to vertical OY axis in bound frame of reference. Nominal alignment of first and third accelerometer axes E_1, E_3 lie in OXZ plane in bound frame of reference.

The model which is also described in [9] embodies a set of most important [1-3, 5-10] factors and parameters that define how errors in sensor measurements form:

1.   Errors of accelerometer misalignment in sensor module frame of reference (bound frame). These are angles θ_1, ϕ_1 and θ_3, ϕ_3 for accelerometers 1 (OX) and 3 (OZ) respectively. θ_1, θ_3 are angles between accelerometer 1 and 3 axes and OXZ plane. ϕ_1, ϕ_3 are angles between projections of E_1 and E_3 on OXZ plane and OX axis (Fig. 1). For accelerometer 2, which corresponds to OY axis, these are angles ν_1 and ν_2 (Fig. 2) – the angles between vertical OY axis and E_2 projection on OXY and OYZ respectively;
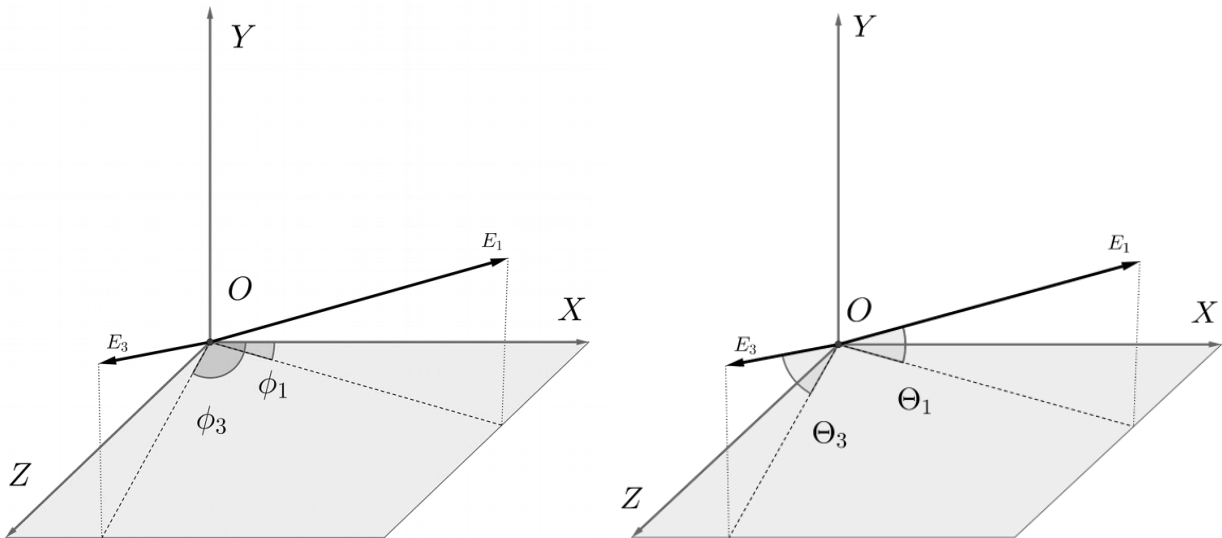

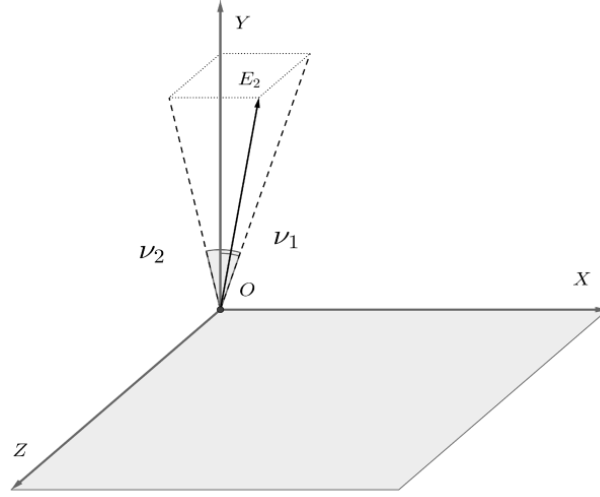Fig.1. Angular parameters for accelerometer 1 and 3 (X and Z) alignment.

Fig. 2. Angular parameters for accelerometer 2 (Y) alignment.

2. $K_{1,2,3}$ - accelerometer 1, 2 and 3 scale coefficients;
3. $B_{1,2,3}$- accelerometer 1, 2 and 3 biases;

Vectors $\boldsymbol{p_1}$, $\boldsymbol{p_2}$, $\boldsymbol{p_3}$ with those parameters as elements are related to accelerometer output by functions:

$$a_i(\boldsymbol{p_i}, t) = K_i \cdot \boldsymbol{E_i}(\theta_i, \psi_i) \cdot \boldsymbol{g}(t) + B_i \ i = 1,3$$
$$a_i(\boldsymbol{p_i}, t) = K_i \cdot \boldsymbol{E_i}(\nu_{1,}\nu_2) \cdot \boldsymbol{g}(t) + B_i \ i = 2 \ . \tag{1}$$

Here:

- $a_i(\boldsymbol{p_i}, t)$- i-th accelerometer output;
- $K_i \cdot \boldsymbol{E_i}$ - i-th accelerometer sensitivity vector determined by sensor alignment and scale coefficient deviation;
- $\boldsymbol{g}(t)$- gravity acceleration vector at particular location, perceived in bound reference frame and time dependent due to rotation of calibrated module and Earth rotation.

Relation between accelerometer alignment parameters and vector $K_i \cdot \boldsymbol{E_i}$ (i = 1, 2, 3) defined by those expressions:

$$\boldsymbol{E_1} = \begin{bmatrix} \sqrt{1 - sin^2\psi_1}\sqrt{1 - sin^2\theta_1} \\ sin\theta_1 \\ -sin\psi_1\sqrt{1 - sin^2\theta_1} \end{bmatrix};$$

$$\boldsymbol{E_2} = \begin{bmatrix} sin\nu_1 \\ \sqrt{1 - sin^2\nu_1 - sin^2\nu_2} \\ -sin\nu_2 \end{bmatrix}; \tag{2}$$

$$\boldsymbol{E_3} = \begin{bmatrix} cos\psi_3\sqrt{1 - sin^2\theta_3} \\ sin\theta_3 \\ \sqrt{1 - cos^2\psi_3}\sqrt{1 - sin^2\theta_3} \end{bmatrix}.$$

Model implies that measurement errors are formed due to deviation of the parameters from their nominal value.

3. Scalar calibration

Scalar calibration method is shown in details in [3, 6, 7, 11] and utilizes scalar function $S(\boldsymbol{p}) = a_1^2 + a_2^2 + a_3^2$ which is a sum of squared output values from each of 3 sensors taken at one moment of time. Vector $\boldsymbol{p}$ is composed of vectors $\boldsymbol{p_1}$, $\boldsymbol{p_2}$ and $\boldsymbol{p_3}$ and holds error model parameters for all accelerometers. Gravity acceleration vector $\boldsymbol{g}$ is a measured reference value in calibration. For defined relative orientation of sensitivity axes and nominal

values of error model parameters one can obtain expected scalar function value $\hat{S}$ and this value will remain constant for every angular orientation of whole module. It is assumed that discrepancy between expected and actual measured values of $S(\boldsymbol{p})$ within small limits is a contributed result of every parameter deviation from their defined nominal value. Assuming these deviations to be small enough, difference $S - \hat{S}$ can be approximated by linear function:

$$S - \hat{S} = \frac{\partial S(\boldsymbol{p_0})}{\partial \boldsymbol{p}} \cdot \delta\boldsymbol{p}, \tag{3}$$

where $\boldsymbol{p_0}$ is a vector of nominal values of error parameters.

With varied angular orientation of accelerometer module equation (3) forms linear equation system where $\frac{\partial S(\boldsymbol{p_0})}{\partial \boldsymbol{p}}$ is dependent of module angular orientation. Note that in scalar calibration method some parameters in $\boldsymbol{p}$ are not observable. The actual vector of parameters available for estimation:

$$\boldsymbol{p} = \begin{bmatrix} \theta_1 + \nu_1 \\ \theta_3 - \nu_2 \\ \psi_3 - \psi_1 \\ K_1 \\ K_2 \\ K_3 \\ B_1 \\ B_2 \\ B_3 \end{bmatrix}. \tag{4}$$

Similar to vector calibration, vector of parameter deviation $\delta\boldsymbol{p}$ in scalar case can be estimated using least square method or Kalman filtering.

Row vectors of partial derivatives $\boldsymbol{D_i} = \frac{\partial S_i(\boldsymbol{p_0})}{\partial \boldsymbol{p}}$ for every moment where measurement is taken form matrix D:

$$D = \begin{bmatrix} \boldsymbol{D_1} \rightarrow \\ \boldsymbol{D_2} \rightarrow \\ \boldsymbol{D_3} \rightarrow \\ \dots \\ \boldsymbol{D_n} \rightarrow \end{bmatrix}. \tag{5}$$

Discrepancies $S_i - \widehat{S_i}$ for every moment of time form vector $\boldsymbol{z}$. Thus calibration problem appears as estimating a solution for inconsistent or over-determined system of linear equations.

$$\boldsymbol{z} = D \cdot \delta\boldsymbol{p}. \tag{6}$$

In case of least square method applied the solution is presented as:

$$\delta\boldsymbol{p} = (D^T D)^{-1} D^T \boldsymbol{z}. \tag{7}$$

The entire effect of estimated deviation of error parameters of the model is contained in vector $\boldsymbol{z}$. However, aside from estimated factors, $\boldsymbol{z}$ is affected by some unaccounted conditions, that form sensor output as well (errors in module angular orientation relative to Earth, thermal distortion). Besides, variations of $\boldsymbol{z}$ will have an effect on accuracy of vector $\delta\boldsymbol{p}$ estimation, and that influence can be evaluated by condition numbercof quadratic matrix $(D^T D)^{-1}$.

Calibration matrix D and, therefore, matrix$(D^T D)^{-1}$ form depending on choices of module angular orientations while taking measurements during calibration process. Orientation sets used earlier yield ill-condition matrices (Table. 1).

The accuracy of estimation is quite high for computer imitation. In this case the error model considers only "clean" $\boldsymbol{z}$ with error factors without any measurement noise and with precise module angular orientations. However, with mixing a noise into $\boldsymbol{z}$ accuracy drops to unsuitable level.

Table 1. Earlier used angular orientation set for scalar calibration.

№	Rotation OX, º	Rotation OY, º	Rotation OZ, º
1	0	0	0
2	40	10	40
3	80	20	80
4	120	30	120
5	160	40	160
6	200	50	200
7	240	60	240
8	280	70	280
9	320	80	320
$cond(D^T D)^{-1}$	3.3·10⁴		

4. Improving conditioning for scalar calibration problem

Even in fixed-base calibration, mounted frame of reference has its errors in angular orientation relatively to Earth. These errors being unaccounted will have their effect on vector of discrepancies. Thus it is important to form well-conditioned system of linear equations, robust to disturbances in **z**, caused by unaccounted factors.

In case of scalar calibration i-th row in matrix D has following form:

$$\boldsymbol{D}_i^T = 2 \cdot \begin{bmatrix} g_{i1}g_{i2} \\ g_{i2}g_{i3} \\ g_{i1}g_{i3} \\ g_{i1}^2 \\ g_{i2}^2 \\ g_{i3}^2 \\ g_{i1} \\ g_{i2} \\ g_{i3} \end{bmatrix}. \tag{8}$$

The approach for conditioning improvement demands determining particular angular orientations so that corresponding rows in calibration matrix has minimal projection on other rows. This approach can be presented as optimization problem (10) for projection function of newly formed row $\overrightarrow{\boldsymbol{D_i}}$ on vector space of other row vectors in D_{i-1} by two angles α and β. These angles define vector of gravity acceleration $g_{bnd}(\alpha,\beta)$ (9) in spherical coordinate system, bound to rotation pivot of module (Fig.3).

$$\boldsymbol{g_{bnd}} = \begin{bmatrix} g_{i1} \\ g_{i2} \\ g_{i3} \end{bmatrix} = \begin{bmatrix} gsin(\alpha)cos(\beta) \\ gsin(\alpha)sin(\beta) \\ gcos(\alpha) \end{bmatrix}. \tag{9}$$

$$minP_{Di}(\alpha,\beta) = ^{\alpha,\beta} \left\| (D_{i-1} \cdot D_{i-1}^T) \cdot (D_{i-1}^T \cdot D_{i-1})^{-1} \cdot \overrightarrow{\boldsymbol{D_i}} \right\|. \tag{10}$$

Here D_{i-1} is a matrix with i-1 rows, formed earlier in the same way. Composing calibration matrix by choosing module orientations according to minimal points of projection function in (10) leads to improvement of the resulting matrix D conditioning.
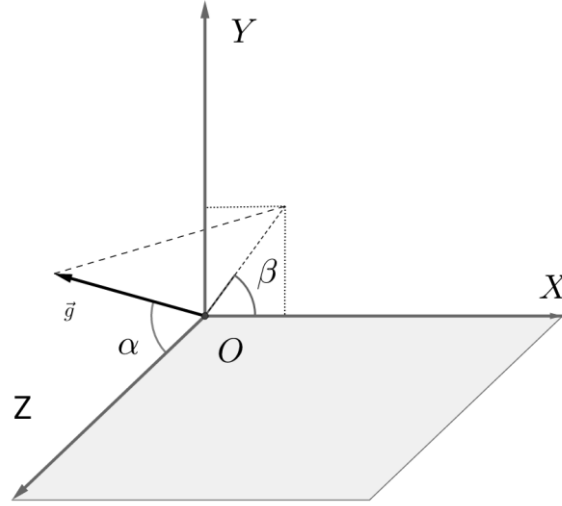
Fig. 3. Vector $\boldsymbol{g_{bnd}}(\alpha, \beta)$ defined in a spherical coordinate system.

The algorithm for composing such calibration matrix is as follows: first measurement of perceived acceleration is taken in initial position of module: alignment of OY axis is vertical, which corresponds to angles $\alpha = -90^o$ and $\beta = 90^o$. Second measurement is taken in such angular orientation, that angles α and β of $\boldsymbol{g_{bnd}}(\alpha, \beta)$ minimize function from (10) with previously calculated row $\boldsymbol{D_1}$ representing the whole calibration matrix. Third measurement is taken in orientation, that minimizes function from (10) for calculated rows $\boldsymbol{D_1}$ and $\boldsymbol{D_2}$. On following iterations newly acquired rows $\boldsymbol{D_i}$ are added to matrix D_{i-1} and projection function is recalculated. Overall, linear system of at least 9 equations must be formed to estimate 9 error parameter deviations.

5. Visualization of a function and verification of the method by computer imitation

Applying optimization algorithm for function $P_{Di}(\alpha, \beta)$ requires preliminary visualization to determine its properties. 3D plots of squared value of function $P_{Di}(\alpha, \beta)$ in area $[0; 2\pi]$ radians of parameters α and β are shown on Fig. 4. Plots are made via CAS Maxima since calculations are done symbolically. The figure includes 8 subsequent plots of squared value of $P_{Di}(\alpha, \beta)$ for each step in which new optimal pair $(\alpha_{min}, \beta_{min})_i$ is found. The complexity of efficiency function increases as new rows $\boldsymbol{D_i}$ are added to calibration matrix. On each of 8 iterations initial points are determined for optimization algorithm using visualization to find $(\alpha_{min}, \beta_{min})_i$, which corresponds to new optimal orientation for module on current iteration i (Fig. 4).

Visualization shows that in area from 0 to 2π radians of both parameters function is periodic on every step and has several minimal points, therefore it is possible to chose several optimal orientations. Initial point for optimization algorithm is picked from visual representation of the efficiency function on every of 8 plots consequently and determine which one of the available minimal points is found. One can also note that after 4th measurement minimal function value jumps to non-zero value showing that minimal possible condition number of calibration matrix D is lower bounded. Adding more angular positions increases minimal function value as well.

The model considered in this paper doesn't account for asymmetry in scaling coefficients, so choosing from multiple points of minima doesn't matter. However, if scaling coefficients are estimated separately for positive and negative projections of perceived acceleration, one can choose a particular area of minima on plot which meets the requirement for module orientation relative to reference acceleration vector.

Acquired set of 9 pairs $(\alpha_{min}, \beta_{min})_i$ is translated into corresponding angles for actual gimbals of sensor module (Table. 2). The scheme of gimbals system is on the Fig. 5.

Table 2. Optimal set of angles of orientation for gimbals.

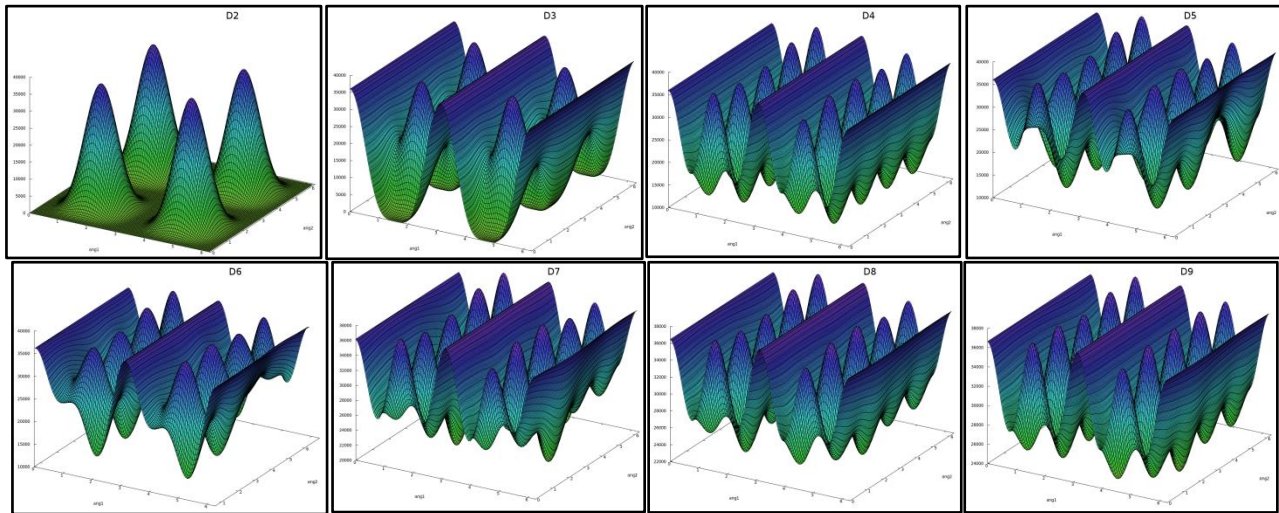| Axes	1	2	3	4	5	6	7	8	9
Z, º	0	90.0074	-90.0000	-136.0061	138.6793	50.5226	135.177	-136.9260	-43.8877
X, º	0	-89.9926	0.5947	35.6601	36.9652	33.2264	-34.3200	-34.5039	36.9528
Y, º	0	90.0000	0.5947	77.0549	-83.1103	-16.0279	73.6480	-76.3967	15.3341
$cond((D^T D)^{-1})$ 263.6104									

Fig. 4. Function $P_{Di}(\alpha, \beta)$ for rows $\boldsymbol{D_i}$, $i = 2..9$.

A computer imitation of fixed base calibration in case of non-optimal (Table 1) and new optimal (Table 2) set of orientations was used to verify the method. Three virtual accelerometer formed an array of gravity vector measurements in a given gimbals angular orientation. The computer imitation designed in such way that sensors output errors are defined by deviation of parameter vector $\boldsymbol{p}$ from its nominal value.

The values of deviation are set in imitation program and provided in Table 3. After receiving measurements from imitation, discrepancy between measured and expected values is calculated in each angular orientation. With these discrepancy values vector $\boldsymbol{p}$ deviation and precision of calibration were estimated.

To compare estimation robustness in both cases the measurements were taken with error in angular module orientation equal for all axes. This error value was random, normally distributed with expected value 0 and variance ranged from 0 to 600 arcsec. The average relative estimation error for 200 calibration imitation repetitions were taken as characteristic of accuracy for single orientation error variance value.
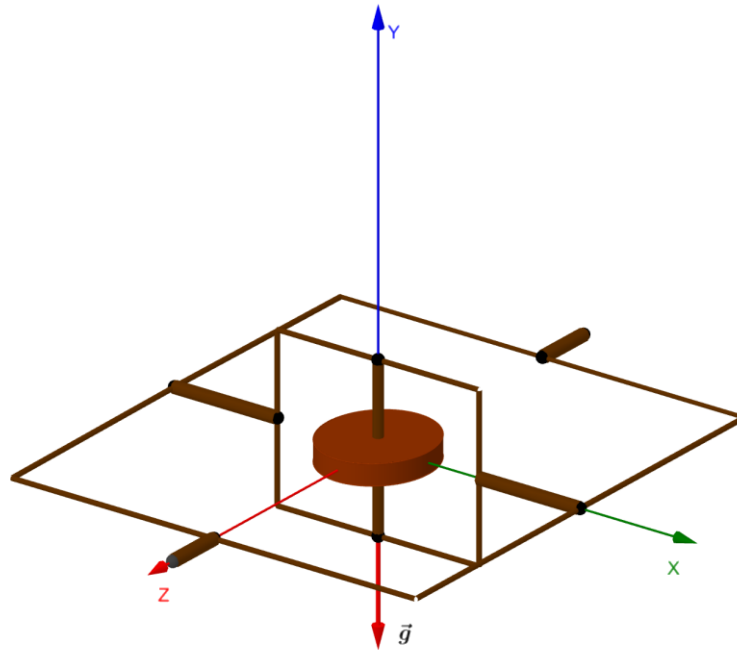
Fig. 5. Scheme of a gimbal system in initial orientation.

The relation between relative estimation error for different calibrated parameters and magnitude of errors in angular positioning of module is shown on Fig. 6 (a, b, c).

Table 3. Nominal values of error model parameters and their deviations in imitation program.

№	Parameter	Value	Deviation value
1	θ_1	0°	10'
2	φ_1	0°	10'
3	v_1	30"	10'
4	v_2	30"	-10'
5	θ_3	0°	10'
6	φ_3	-90°	-10'
7	K_1	1	0,5%
8	K_2	1	0,5%
9	K_3	1	0,5%
10	b_1	0	10^{-3} m/s²
11	b_2	0	10^{-3} m/s²
12	b_3	0	10^{-3} m/s²

These plots show that scalar calibration performed with optimal set of module orientations is in general more robust to errors of angular positioning of module during the calibration. Fig 6 (a) shows that in case of using optimal set of angular positions estimation accuracy for parameters $\theta_1 + v_1$ and $\theta_3 - v_2$ are not affected by errors in module orientation in margin of 0 to 600 arcsec of mean error value, while in case of using a non-optimal set a relative estimation error grows in linear manner in same margin. The relative estimation error of parameter $\psi_3 - \psi_1$ in both cases doesn't grow with increasing magnitude of errors in module orientation. In overall, the relative error of estimation of all angular parameters $(\theta_1 + v_1, \theta_3 - v_2, \psi_3 - \psi_1)$ using optimal set of orientations is about 0.86%.
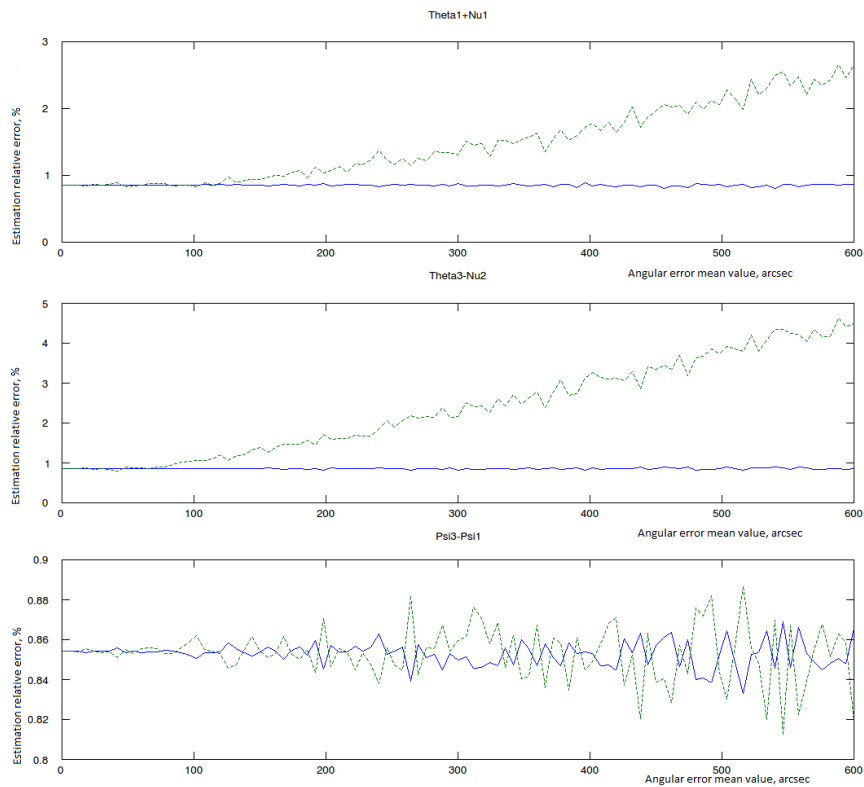
Fig. 6 (a). The relation between relative estimation error for deviation of sensor alignment parameters and magnitude of errors in angular positioning of module for non-optimal (dashed) and optimal set of module orientations.
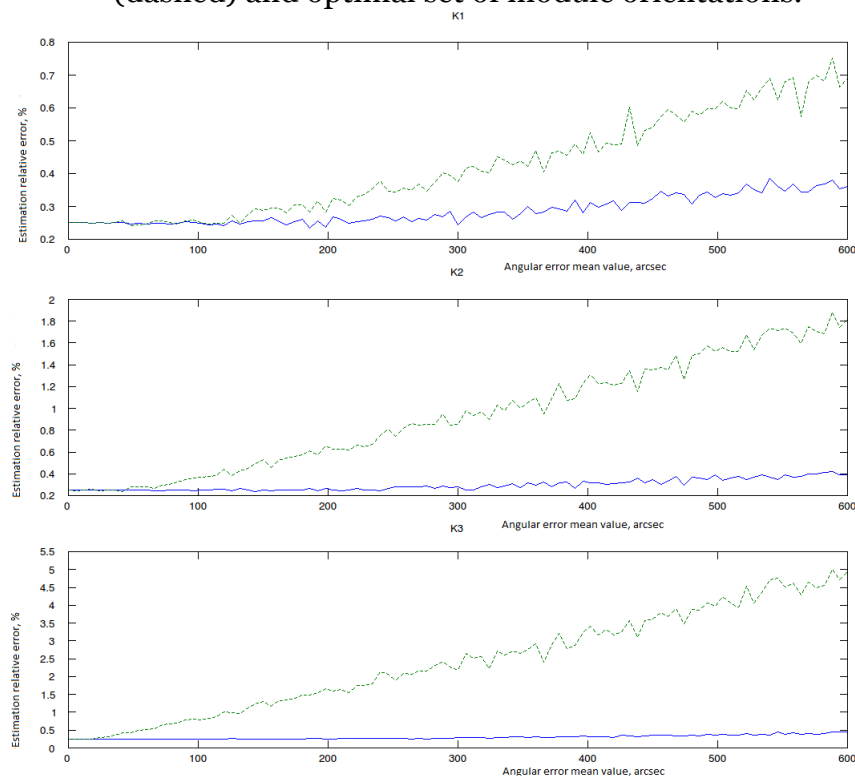

Fig. 6 (b). The relation between relative estimation error for deviation of sensor scaling co-efficients and magnitude of errors in angular positioning of module for non-optimal (dashed) and optimal (solid) set of module orientations.
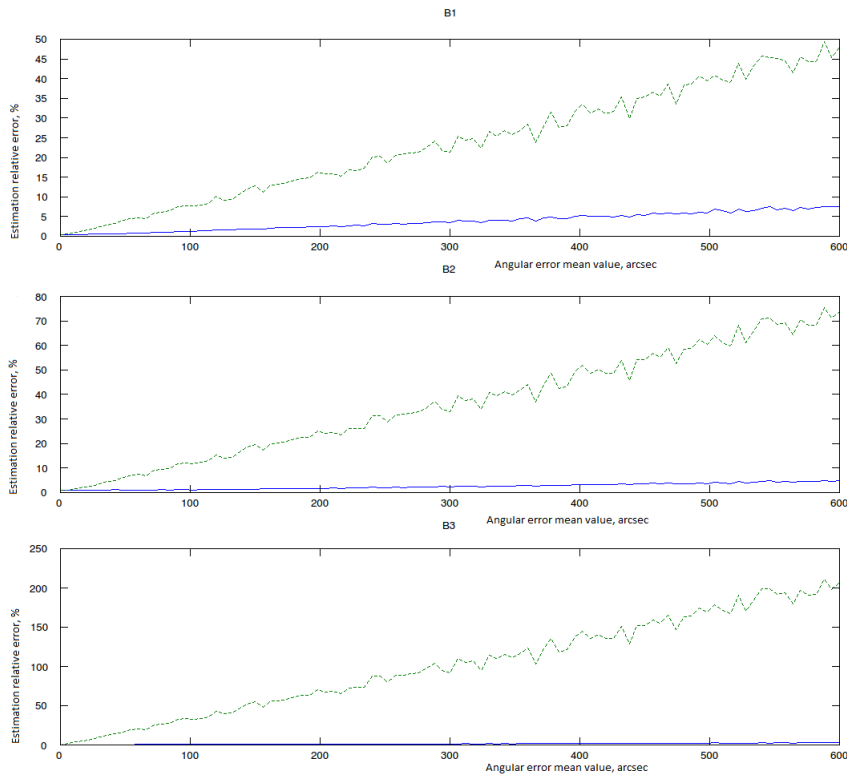
Fig. 6 (c). The relation between relative estimation error for deviation of sensor bias and magnitude of errors in angular positioning of module for non-optimal (dashed) and optimal (solid) set of module orientations.

Fig. 6 (b) and (c) show that relative errors for scaling coefficients and biases are less robust to accuracy of module alignment. In both cases these estimation errors grow linearly with mean module alignment error. Note, that estimation accuracy of same parameters for different accelerometers have different susceptibility to module alignment errors, but in case of using optimal set this susceptibility becomes similar. Also, with optimal set of orientations it is 2-10 times weaker than with non-optimal set for scaling coefficients and 7-27 times weaker for biases. In overall, with mean module alignment error of 600 arcsec, the relative estimation error for scaling coefficients is about 0.4% and 7.5% for biases.

6. Conclusion

Calibration of accelerometer module is an operation performed to determine the parameters of model, which describes forming of measurement errors. In calibration on fixed base unaccounted factors such as errors in module angular positioning lead to losses in calibration accuracy. The effect of unaccounted factors on accuracy depends on problem conditioning which, in its turn, is determined by used set of module orientation. The paper present an approach for scalar calibration case to chose a module set of orientations that is optimal in terms of problem conditioning. An approach includes visual representation of function which characterize an efficiency of calibration measurement made in each angular module position for estimation of solution vector, the minimization of each function and transformation of minimal point to Euler angles for gimbal positioning.

The method verified in computer simulation by comparison of accuracy using optimal and nonoptimal sets of module orientations. The imitation of both cases was performed in presence of errors in angles of module orientation in range from 0 to 600 arcsec. With optimized set of calibration orientations the estimation accuracy for angular parameters (accelerometer alignment errors inside a module) is non-susceptible to errors in module alignment during measurements, while in case of using a non-optimized set the estimation accuracy of only one angular parameter is not affected by these module alignment errors.

Relative errors of estimation for scaling coefficients and biases grow linearly with module alignment errors for both cases. However with optimized set of orientations this growth is 2- 27 times slower in comparison with estimations done with non-optimized set.

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