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Stereo images of error surfaces in problems of numerical methods verification

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Abstract

The problems of verifying numerical methods and their comparative analysis are an important and relevant section of computational mathematics. Modern technologies make it possible to compare numerical methods not for one single task, but for a class of problems using parametric studies implemented in the form of a generalized computational experiment.

Such solutions for two-parameter problems are presented in the form of error surfaces constructed for each numerical element participating in the comparison. The visual representation of such surfaces in stereo mode gives the researcher the possibility of a deep and thorough comparative analysis. This task is the application of previously developed methods and approaches to creating stereo images and texts to visualize the results of solving the problem of falling supersonic flow onto a flat plate at an angle of attack.

This problem for inviscid flows has an exact solution, which is used as the standard for comparison. A parametric study is carried out with a variation of two parameters - the angle of attack and the Mach number of the flow. The results of constructing error surfaces for various solvers of the open software package OpenFOAM are presented. The construction of error surfaces in stereo mode is implemented for an autostereoscopic monitor based on a multi-view method.

Keywords: numerical methods, verification, error surfaces, parametric studies, stereo images, autostereoscopic monitor, multi-view.

1. Introduction

The tasks of verifying numerical methods and their modifications are an actual section of modern computational mathematics. This is confirmed by the introduction in 2018 of the Federal Standard for the numerical simulation of supersonic inviscid gas flows and software verification [1]. Such standards will determine the direction of research in this area over the next decade. Verification problems were solved throughout the development of numerical methods.

However, now due to the development of high-performance computing tools, new verification opportunities are emerging. Earlier, the verification of the numerical method was carried out for separate problems, and in the case of a positive result the obtained computational properties of the considered numerical method extended to similar problems by default.

At present, parallel technologies make it possible to solve parametric problems in multitasking mode and, thus, obtain a comparison of accuracy not only for a single task, but for a whole class of problems determined by a variation of key parameters.Such an approach corresponds to the concept of a generalized computational experiment being developed in Keldysh Institute of Applied Mathematics [2-4]. The construction of a generalized computational experiment for comparative analysis of the accuracy of several numerical methods, where the number of considered numerical methods is another key parameter, allows us to develop approaches for assessing accuracy in the absence of a reference solution (known exact solution or experimental data). Such an approach is the estimation of accuracy on an ensemble of numerical solutions [5,6].

This work uses as a basis for constructing stereo images the numerical results obtained using the generalized computational experiment in [7,8]. In these papers, a class of computational gas dynamics problems is considered that describe the incidence of an inviscid supersonic gas flow onto a flat plate at an angle of attack.

With such a fall, an oblique shock wave is formed. The Mach number and angle of attack are used as key parameters. These values vary in certain ranges. This problem has an exact solution. Numerical results are compared with this exact solution at each point of the calculation domain. For each combination of key parameters, an error is evaluated in the norm of L1 and L2. The obtained results make it possible to construct an error surface as a function of two key parameters of the problem.

Similar calculations for several numerical methods implemented in the solvers of the open software package OpenFOAM, make it possible to build several such surfaces on one frame. This opens up the possibility of a deep and clear comparative analysis of the accuracy for considered numerical methods.

The construction of such a generalized computational experiment involves the creation of a unified computing technology from solving a direct problem up to visual analysis of the results.

One of the most expressive and demonstrative forms of visualization is the construction of stereo animations presenting the results of numerical studies. The works [9–16] reflect a wide range of applications of modern stereo systems and the development of practical algorithms for stereo representation of static and animated images.

This topic also includes a series of researches carried out in Keldysh Institute of Applied Mathematics [12, 17-19, 20-23]. These researches are based on the available stereo systems of two types.

The first type of device is a 3D projection stereo system for displaying stereo presentations, graphics and stereo films. It is an example of a classic stereoscopic system using two projectors, a screen and linear polarization glasses. The papers [20-21] describe in detail studies on the presentation of textual information on this type of stereo device using a linear stereo base. In these works, test inscriptions were built with variations of the font, background, font embossment, the whole angle of rotation of the inscription, and shift along the linear stereo base. The parameters were found that provide the most bright stereo effect. Basic requirements for the fonts used and a number of special conditions were identified. The satisfaction of these conditions is necessary to achieve the optimal result.

The second type of device is the Dimenco DM654MAS autostereoscopic monitor. Autostereoscopic monitors provide glasses-free stereo imaging. There are two ways to demonstrate objects: either using a composite frame containing views of the visualization object at different angles that form a certain viewing sector - this is called a multi-view method, or using depth maps.

In previous works devoted to stereo images of textual information [20-22], a series of computational experiments were described for stereo systems of both types — classical stereo systems using special glasses and an autostereoscopic monitor. During the experiments, when constructing the inscription, various parameters varied: font size, rotation angle on each frame in multi-view, and also the distance between frames for a linear stereo image.

After solving the problem of creating stereo texts on their own, as separate frames in a stereo presentation or stereo animation, another important sub-task was identified: combining images and text information in one frame. This problem is described in [23] in detail. The developed methods and approaches for constructing stereo images made it possible to apply them to the construction of error surfaces for the analysis of the comparative accuracy of

OpenFOAM solvers for the oblique shock wave problem with variation of the Mach number and angle of attack. To build stereo images, a multi-view method was used on an autostereoscopic monitor.

2. The construction of stereo images on an autostereoscopic monitor using multi-view method

An autostereoscopic monitor has the ability to demonstrate a visualization object using a composite frame containing views of the visualization object at different angles, which together form a certain viewing sector. Usually, these are nine views. These nine views form eight stereopairs ([1 | 2], [2 | 3], [3 | 4], ..., [8 | 9]), however, the viewer can observe only one of the stereopairs in one position, depending on the location in defined angular sector of observation. When the observer moves from sector to sector, he receives information about the object using all nine views, that is, as if looking at the visualization object from different angles.

When building a multi-view stereo image, nine frames are combined into one image according to the principle of a 3 × 3 matrix. In the first and last (ninth) frames, the represented object is in its extreme positions. In the first frame, the object is in its rightmost position, and in the last frame, in the leftmost position. Then in the middle frames, respectively, from the second to the eighth, with certain equal intervals, it moves to the left until it reaches the extreme left position. In this case, in each frame, the object is progressively rotated by the same angle and shifted by a predetermined interval. This allows you to achieve the most effective result when creating a stereo image of the object.

Figure 1 shows one of the results of previous studies - a multi-view image of the simulation results of a supersonic flow around a cone at an angle of attack with the corresponding inscription [23]. The task was to combine in one stereo image the objects of different depths and sizes and text information (in this case, the cone and the inscription at the figure). Here the image of the simulated cone and separately the inscriptions to it are combined. Each of them is rotated by its own experimentally revealed angle and is shifted by a certain distance, also different. As shown in the figure, a matrix of images is further compiled, which in turn comprise a single stereo image. In the end, the inscription was located on top of the cone, but behind its tip, which in turn was perceived by observers as protruding from the screen by several centimeters.

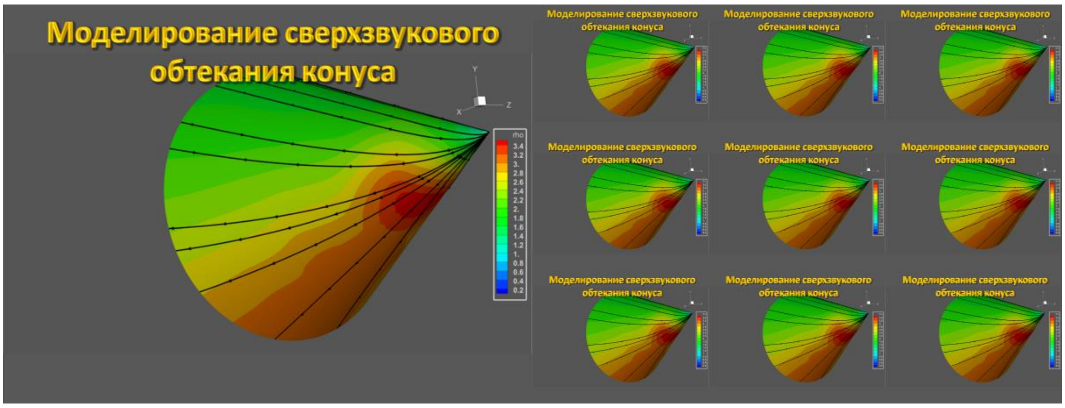

Fig. 1. Image of the simulation results of a supersonic flow around a cone with the corresponding signature [23].

A similar problem was posed for constructing stereo mappings of error surfaces with corresponding inscriptions.

3. The construction of stereo for the problems of comparing the accuracy of numerical methods

This section presents the results of constructing error surfaces for four OpenFOAM solvers with variations in the Mach number from 2 to 4 and variations in the angle of attack from 6 to 20 degrees [8]. It should be noted that error surfaces for the class of problems of the numerical methods accuracy comparative analysis were constructed in [8] for the first time. The left part of Figure 2 shows four error surfaces for fourOpenFOAM solvers - rhoCentralFoam (rCF), pisoCentralFoam (pCF), sonicFoam (sF) and QGDFoam (QGDF) with the corresponding text labels and notations. To construct a single stereo image, the same techniques were used as in [23] for the image of the cone. The constructed multi-view image is displayed on the right side of Figure 2.

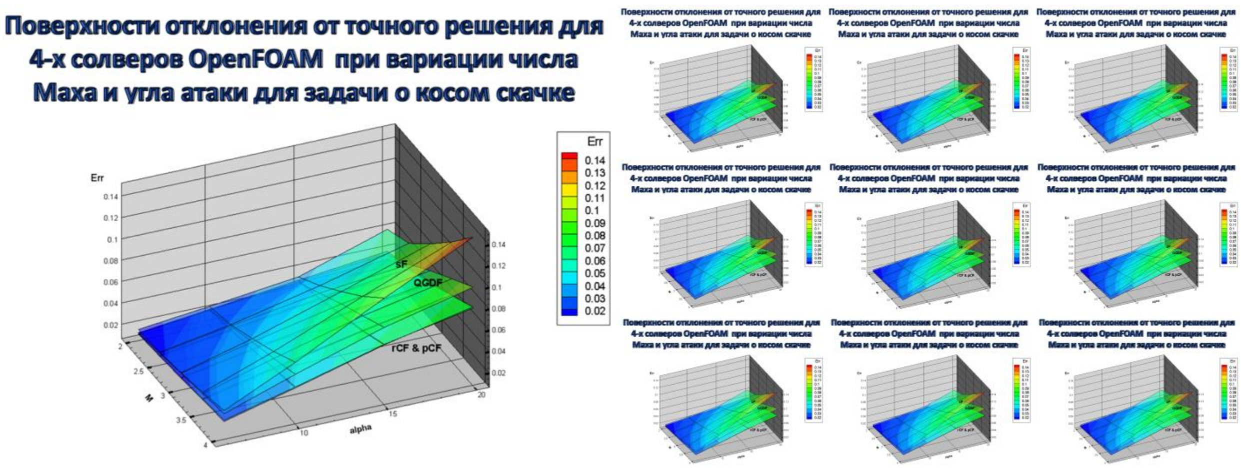

Fig. 2. Image of the surface deviation from the exact solution for four OpenFOAM solvers with variation of the Mach number and angle of attack for the oblique shock wave problem.

Figure 2 shows that the best accuracy in the class of problems is provided by the rCF and pCF solvers, for which the error surfaces almost coincide. The developed technology allows one to create stereo images for these surfaces separately. The results are presented in Figure 3.

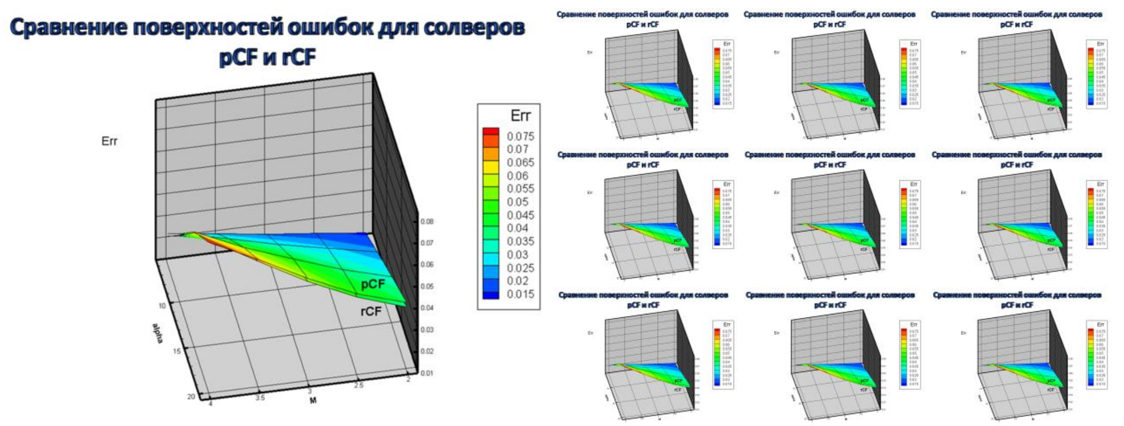

Fig. 3. Image of error surface comparison for pCF and rCF solvers.

Figure 3 shows that the divergence of error surfaces is most pronounced for the smallest Mach number and the largest angle of attack. In Figure 4, this area is presented in close-up.

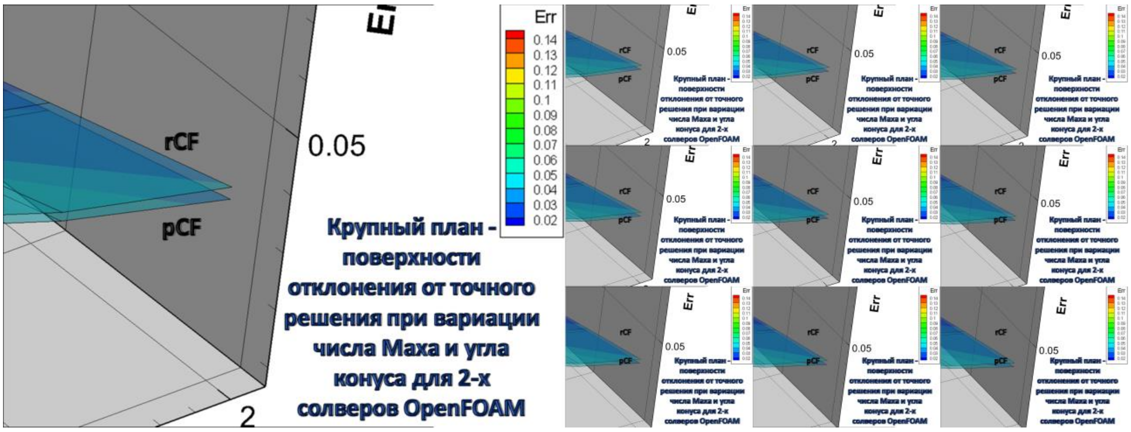

Fig. 4. A close-up image of the surface deviation from the exact solution when the Mach number and cone angle are varied for two OpenFOAM solvers.

4. Conclusion

This work continues a series of studies devoted to the implementation of the project to create stereo presentations of mathematical modeling problems results. The results of numerical experiments on construction of error surfaces for various OpenFOAM solvers are presented. The error surfaces are constructed on an autostereoscopic monitor using the multi-view representation. The experiments were carried out for a comparative analysis of the numerical methods accuracy. The accuracy was compared using the classical oblique shock wave problem, which has an exact solution. The construction of stereo frames is carried out in the previously developed mode of combining in one stereo frame the main object of the visualization and the corresponding text labels and symbols. The constructed stereo frames provide the researcher with the possibility of a deep and thorough visual analysis of the results.

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