

Visualization Metaphors for Fuzzy Cognitive Maps

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Abstract

At present, a cognitive approach is widely used for modeling and decision support in semi-structured systems. This approach focuses on the development of formal models and methods supporting the intelligent problem-solving process as they take into account human cognitive capabilities (perception, conception, cognition, understanding, explanation). In general terms, a cognitive model is thought of as a model of expert knowledge about a system, processes occurring in it and laws and principles of its functioning represented as a cognitive map. A causal network which reflects researcher's subjective notion of the system as a number of semantic categories known as factors or concepts and a set of cause-and-effect relations between them is referred to as a cognitive map.

One of the key ideas in effective dealing with a cognitive model is providing its visual representation. In the paper, a visualization metaphor for fuzzy cognitive maps is proposed and its two components, namely spatial metaphor and representation metaphor, are defined. Graphs visualization algorithms and cognitive clarity notion form the basis for constructing a visualization metaphor. To assess the level of cognitive clarity, a set of criteria is proposed.

The mechanism of interactive control of a visual image of a fuzzy cognitive map in the cognitive modeling support system IGLA is considered. This mechanism provides flexible adjustment of representation metaphors visual characteristics, which allows the researcher to focus on the aspects of a cognitive model that are of most interest to him at a particular stage of the analysis.

The possibilities of using visualization metaphors and interactive control of the fuzzy cognitive maps visual representation in IGLA system are illustrated by the example of a cognitive model of decision support in tooling design.

Keywords: fuzzy cognitive map, graph visualization, cognitive clarity, visualization metaphor.

1. Introduction

The cognitive approach is one of approaches widely used to modeling semi-structured systems and making controlling such systems decisions. According to the definition given in [2], this approach focuses on the development of formal models and methods supporting the intelligent problem-solving process as they consider human cognitive capabilities (perception, conception, cognition, understanding, explanation). Structure and target modeling, as well as simulation modeling methods based on cognitive approach are commonly subsumed under the umbrella term “cogni-

tive modeling”. In general a cognitive model is thought of as a model of expert knowledge about a system, processes occurring in it, laws and principles of its functioning that are represented as a cognitive map. A causal network reflects researcher's subjective notion of the system (individual or collective) as a number of semantic categories known as factors or concepts and a set of cause-and-effect relations between them is referred to as a cognitive map. A cognitive model is an effective tool for exploratory and estimative analysis of the situation. It does not give an opportunity to obtain accurate quantitative characteristics of the system under study,

but it allows to assess trends related to its functioning and development, and to identify the key factors influencing these processes.

As stated in [9], it is believed that knowledge of a crude, perhaps even a hypothetical model of a system allows us to predict development scenarios of initial situations under different control actions by varying the model variables. This makes it possible to search and generate effective solutions to control the system, as well as to identify risks and develop strategies to reduce them.

Cognitive modeling starts with creating a cognitive map of a system under study based on information received from experts or analysis of the available system data [15]. At the next stage, the simulation takes place directly. Its main objectives are forming and testing hypotheses about the structure of the system under study that explain its behavior; developing behaviour strategies for the specific situation in order to achieve target states.

Problems solved by cognitive modeling can be divided into two groups:

- static (structure and target) analysis, which goals are finding the key factors influencing the targets most, identification of contradictions between the targets, feedback loops analysis, etc.;
- dynamic (scenario) analysis aimed at prediction of system states under various control actions and search for control solutions bringing the system to the target state.

Fuzzy logic is most commonly used as mathematical apparatus to represent and analyze cognitive models. There is a whole class of cognitive models based on different types of fuzzy cognitive maps (FCMs). A detailed overview of such models can be found, for instance, in monograph [3]. One of FCM varieties well-proven in practical problems of analyzing and modeling semi-structured social, organizational, and economic systems is V.B. Sylov's fuzzy cognitive maps [11, 14].

One of the key ideas of effective application of a cognitive model is to provide its visual representation. The paper proposes a visualization metaphor for fuzzy cognitive

maps (by the example of Sylov's FCM) which is based on graph visualization algorithms and the concept of cognitive clarity. Examples of application of various visualization metaphors and the possibility of interactive control of a FCM visual image in IGLA decision support system developed with the participation of the authors are presented. A more detailed description of IGLA system can be found in [10] and a demo version can be found at <http://iipo.tu-bryansk.ru/quill>. The system is a Windows application based on Microsoft .NET Framework, and a network multi-user version is currently being developed.

2. Formal definition, structure and analysis methods of Sylov's fuzzy cognitive map

As previously mentioned, the cognitive model is based on formalization of cause-and-effect relations which occur between factors characterizing a system under study. The result of the formalization represents the system in the form of a cause-and-effect network, termed a cognitive map and having the following form:

$$G = \langle E, W \rangle,$$

where $E = \{e_1, e_2, \dots, e_K\}$ is a set of factors (also called concepts), W is a binary relation on the set E , which specifies a set of cause-and-effect relations between its elements.

Concepts can specify both relative (qualitative) characteristics of the system under study, such as reliability, producibility, and absolute, measurable values – time, labour intensity, cost, etc. Moreover, every concept e_i relates to a state variable v_i , which specifies the value of the corresponding index at a particular instant. State variables can possess values expressed on a certain scale, within the established limits. Value $v_i(t)$ of state variable at instant t is called the state of concept e_i at the given instant. Thus, the state of the simulated system at any given instant is described by the state

of all concepts included in its cognitive map.

Concepts e_i and e_j are considered to be connected by relation W (designated as $(e_i, e_j) \in W$ or $e_i W e_j$) if changing the state of concept e_i (cause) results in changing the state of concept e_j (effect). In this case we say that concept e_i influences concept e_j . Besides, if the value increase of the concept-cause state variable leads to the value increase of the concept-effect state variable, then the influence is considered positive (“strengthening”); if to the decrease – then negative (“inhibition”). Therefore, the relation W can be represented as a union of

two disjoint subsets $W = W^+ \cup W^-$, where W^+ is a set of positive relations and W^- is a set of negative relations.

Fuzzy cognitive model assumes that the influence between concepts may vary in intensity, whereas, intensity may be constant or variable in time. Considering this assumption, W is set as a fuzzy relation, however, its setting depends on the adopted approach to formalization of cause-and-effect relations. A cognitive map with fuzzy relation W is termed a fuzzy cognitive map. Sylov’s fuzzy cognitive map represents FCM, characterized by the following features.

1. State variables of concepts can possess values on the interval $[0, 1]$.
2. Influence intensity is considered constant, so relation W is specified as a set of numbers w_{ij} , characterizing the direction and degree of influence intensity (weight) between concepts e_i and e_j :

$$w_{ij} = w(e_i, e_j),$$

where w is a normalized index of influence intensity (characteristic function of the relation W) with the following properties:

- a) $-1 \leq w_{ij} \leq 1$;
- b) $w_{ij} = 0$, if e_j does not depend on e_i (no influence);
- c) $w_{ij} = 1$ if positive influence of e_i on e_j is maximum, i.e. when any changes in the system related to concept e_j are uniquely determined by the actions associated with concept e_i ;
- d) $w_{ij} = -1$ if negative influence is maximum, i.e. when any changes related to concept e_j are uniquely constrained by the actions associated with concept e_i ;
- e) w_{ij} possesses the value from the interval $(-1, 1)$, when there is an intermediate degree of positive or negative influence.

FCM of this structure can be graphically represented as a weighted directed graph. The points correspond to elements of set E (concepts) and arcs correspond to nonzero elements of relation W (cause-and-effect relations). Each arc has a weight which is specified by the corresponding value w_{ij} . In this case, relation W can be represented as a matrix of dimension $n \times n$ (where n is the number of concepts in the system), which can be considered as the graph adjacency matrix and is termed a cognitive matrix.

Fig. 1 shows an example of a fuzzy cognitive map of choice of a stamping manufacturing technology [8]. This FCM was built within the framework of the decision support model in the field of computer-aided production tooling design developed with direct participation of its authors. The colour of the arc sets the sign of the corresponding cause-and-effect relation between the concepts: red means a positive relation, blue indicates a negative one, and the thickness of the arc determines the intensity of the relation.

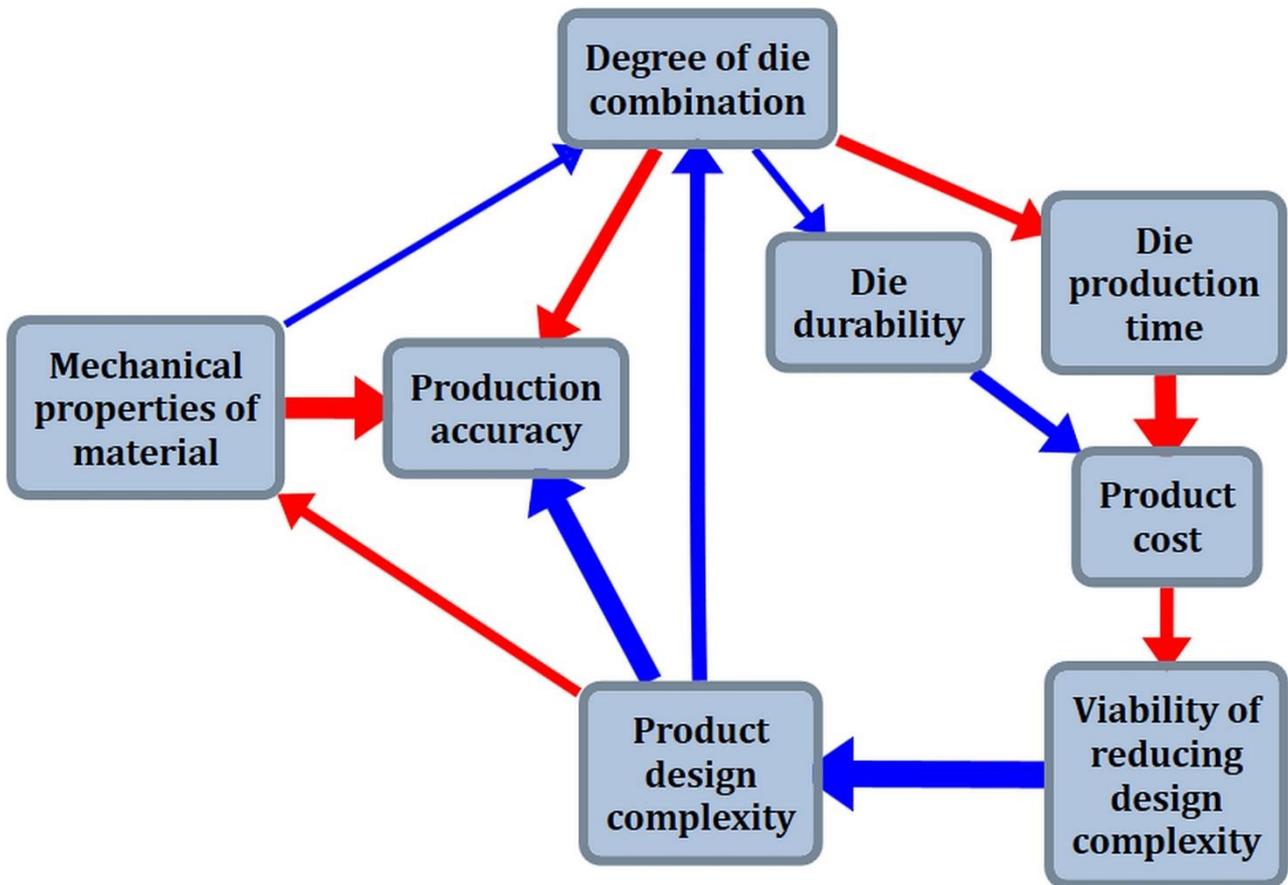


Fig. 1. Fuzzy cognitive map for the problem of choosing a stamping manufacturing technology

Fig. 2 shows a diagram of a generalized algorithm for constructing and analyzing a cognitive model of a semi-structured system based on Sylov's FCM. In the present paper, the stage of FCM visualization is studied. A more detailed description of the other stages of this algorithm and methods used at them can be found, for example, in [8, 10, 11, 14, 15].

3. Fuzzy cognitive maps visualization problem

Visual analysis is an integral part of the cognitive modeling process, both at the stage of constructing a cognitive map and at the stages of its verification and evaluation. The importance of clear and user-friendly visual representation of the cognitive map is conditioned by the following circumstances.

1. A visual representation of a cognitive map often provides the researcher (expert, analyst) with the only opportunity to "grasp the model at a glance," with the result that he can quickly notice errors (for

example, gaps or redundancy) made when constructing it.

2. A visual representation is essential for presentation of cognitive map verification results (at its simplest, verification is a search for redundant transitive ways to transfer influence between concepts and closed infinitely reinforcing or weakening influence cycles).

3. A visual representation can be used to present the results of a structure and target and scenario analyses of a cognitive model. However, in publications on cognitive modeling, the problem of visualization of cognitive maps has received little attention. Certain aspects of this problem were considered in [1, 12]. In previously published paper [6], an approach was proposed that implies reduction of Sylov's FCM visualization problem to the graph visualization task. Several graph visualization algorithms were investigated. In accordance with the idea underlying construction and operation of these algorithms, we can distinguish algorithms based on physical analogies (force algorithms), algorithms based on self-organizing principles, and

algorithms for level-by-level representation of directed graphs.

The most suitable for implementation in IGLA system were acknowledged according to the results of experimental verification of the considered algorithms in terms of real FCMs, the LinLog algorithm [17], belonging to the class of force algorithms, and the ISOM algorithm [16], based on self-organizing principles. . A further development of this approach is its expansion to the visualization metaphor in accordance with the visual representation cognitive clarity requirements.

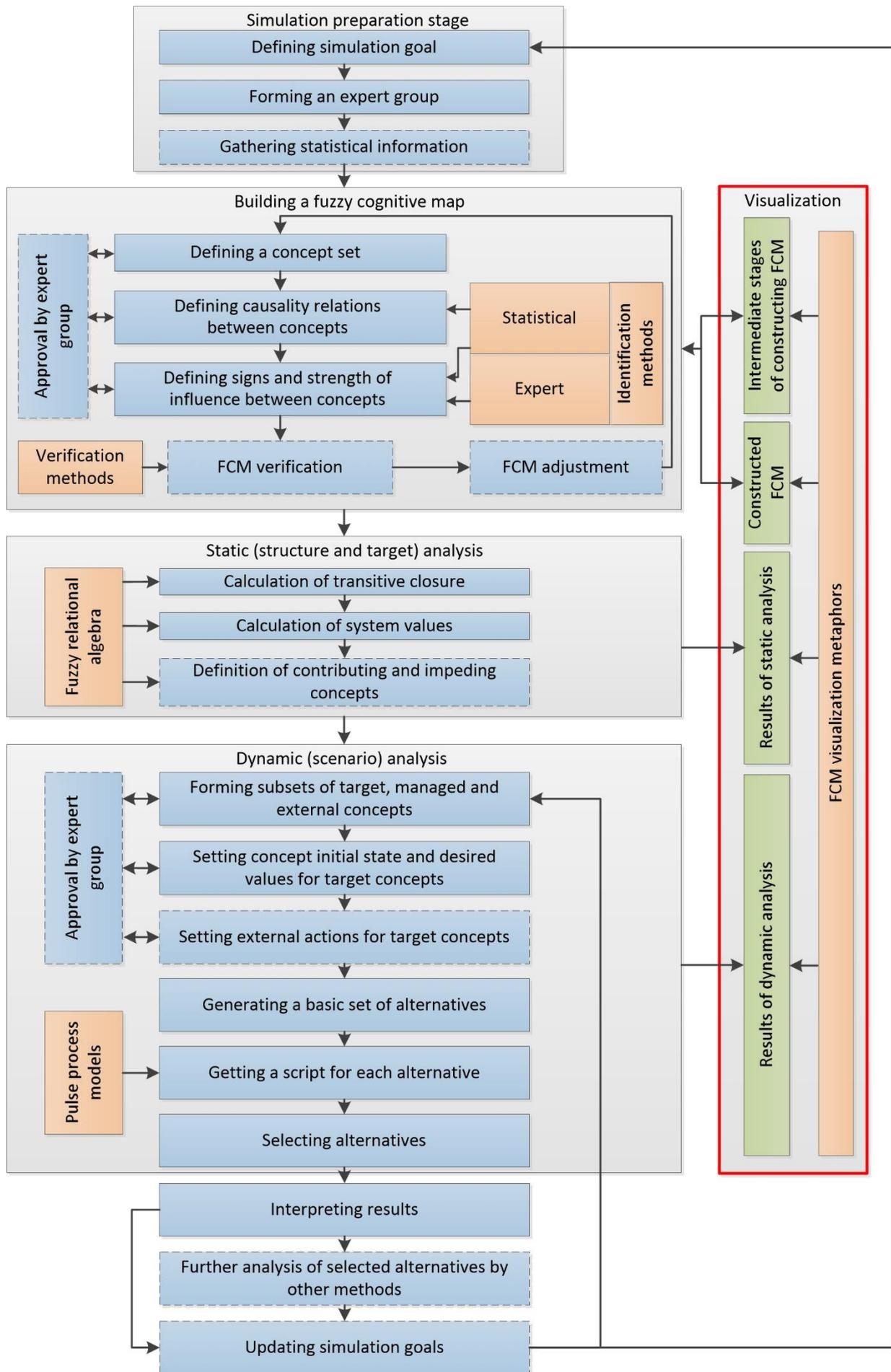


Fig. 2. Generalized algorithm for constructing and analyzing a cognitive model

4. Constructing visualization metaphor

In [5], visualization metaphor is defined as mapping of a visualized object from the data space of the original task into the object of the representation space, which occurs by conditional transfer of the elements attributes of one set to elements of another set.

Moreover, according to [5], visualization metaphor can be used at two stages of solving visualization problems. The first time is when moving from the source data to the space of the visual model. For this case, the term "spatial metaphor" is used. For the second time, the need to apply this technique arises at the stage of clarifying the results of visualization, strengthening the components necessary for problem solving, filtering redundant components of the image created, etc. For this case, the term "representation metaphor" is used. The actual role of the spatial metaphor is to transform current problem data into data describing space elements of the visual model. Representation metaphor is a means of uncovering potential benefits of an already applied spatial metaphor.

Let us consider the proposed method of constructing two components of a FCM visualization metaphor – spatial metaphor and representation metaphor.

Since spatial metaphor implies transition to the visual model space, it is necessary to determine the appearance of this space first – in particular, its dimension.

In [12], a "cognitive cloud" model (that is actually a spatial metaphor), implying the location of a cognitive map in three-dimensional space, is presented. It is hypothesized that such visual representation will contribute to a greater readability of a cognitive model (compared to the location of the map on a plane). This hypothesis is supported by the example of application of the proposed approach.

However, metaphor based on the "cognitive cloud" model is not universal, since it focuses on visualization of cognitive maps of a specific structure (namely, with the presence of pronounced "factor-cores"),

while the potential effectiveness of this metaphor for visualizing unstructured maps is questionable. Besides, visualization in three-dimensional space is obviously more resource-intensive than in two-dimensional one, which can negatively affect the speed of model rendering and its response time to user actions, especially with many concepts and cause-and-effect relations. Thus, it seems appropriate to develop such a metaphor in a two-dimensional space (on a plane), which, on the one hand, is free from the drawbacks and complications of a three-dimensional metaphor, and on the other hand, is more universal and suitable for an unstructured FCM.

As previously mentioned, the FCM visualization problem is generally reduced to the graph visualization problem, which, in a two-dimensional case, can be solved with the help of an extensive class of graph tilting algorithms. Thus, the spatial metaphor of FCM visualization should be based on these algorithms. However, the problems of human limited cognitive abilities when reading graphs should also be considered (a detailed analysis of this problem can be found, for example, in [13]). The approach proposed in the above-mentioned work [6] allows obtaining satisfactory visualization results, but it ignores an important aspect of the resulting image quality, namely, achieving its cognitive clarity. The concept of cognitive clarity and related criteria will be discussed further.

Application of the spatial metaphor to the source data (cognitive graph structure) allows to obtain the optimal in some respects location (i.e. coordinates on the plane) of its points and arcs, which is. Development and description of specific criteria for the optimal location is beyond the scope of this paper and is the direction for further research.

As stated above, the representation metaphor is intended to "reveal the advantages" of the spatial metaphor being used, which results in a transition from the visual model of the object under study to its visual image. Based on the multi-stage process of cognitive modeling (which includes construction of the model and its verification,

as well as various types of its analysis), it's necessary to develop several different representation metaphors. Moreover, each of these metaphors must correspond to a certain stage of modeling and contribute to the achievement by the researcher of the goal set at this stage. Thus, representation metaphors should be built on the basis of the researcher's perception emphases at a particular stage of cognitive modeling.

Let us enumerate and exemplify the main FCM representation metaphors implemented in IGLA system. We will use the previously mentioned cognitive map of choice of a stamping manufacturing technology (Fig. 1).

1. The main metaphor that is used "by default" corresponds to the cognitive map that presented on Fig. 1. This representation metaphor allows the researcher to focus on the structure of the cognitive map as a whole, without being distracted by features of its individual concepts. For this metaphor, a single colour (gray-blue) is used to represent all concepts. As noted above, the colour of the arc sets the sign of the corresponding cause-and-effect relation between concepts: red means a positive relation ("strengthening") while blue means a negative relation ("inhibition"). The thickness of the arc determines relation intensity.
2. Metaphors implying visual differentiation of concepts by their types or their

belonging to certain semantic groups, which are set by experts themselves when building a cognitive model. Fig. 3 shows an example of visual differentiation for focusing researcher's attention on the relative position of managed (yellow), observed (green) and target (gray-blue) concepts. Note (following [10]) that managed are concepts the state of which is directly manageable; observed are concepts which state cannot be defined directly and is determined by the state changes of concept-causes; the target ones are the concepts that need to be brought into a given target state. So, in the example under consideration it is assumed [8] that:

- when choosing a stamping manufacturing technology, the designer can directly influence such factors as mechanical properties of material, product design complexity and the degree of die combination;
- the target states of the task of choice are sufficient due to durability and high level of production accuracy while retaining the required level of product design complexity;
- finally, such parameters as die production time and product cost can not be influenced directly, and their values depend on the state of the target and managed factors.

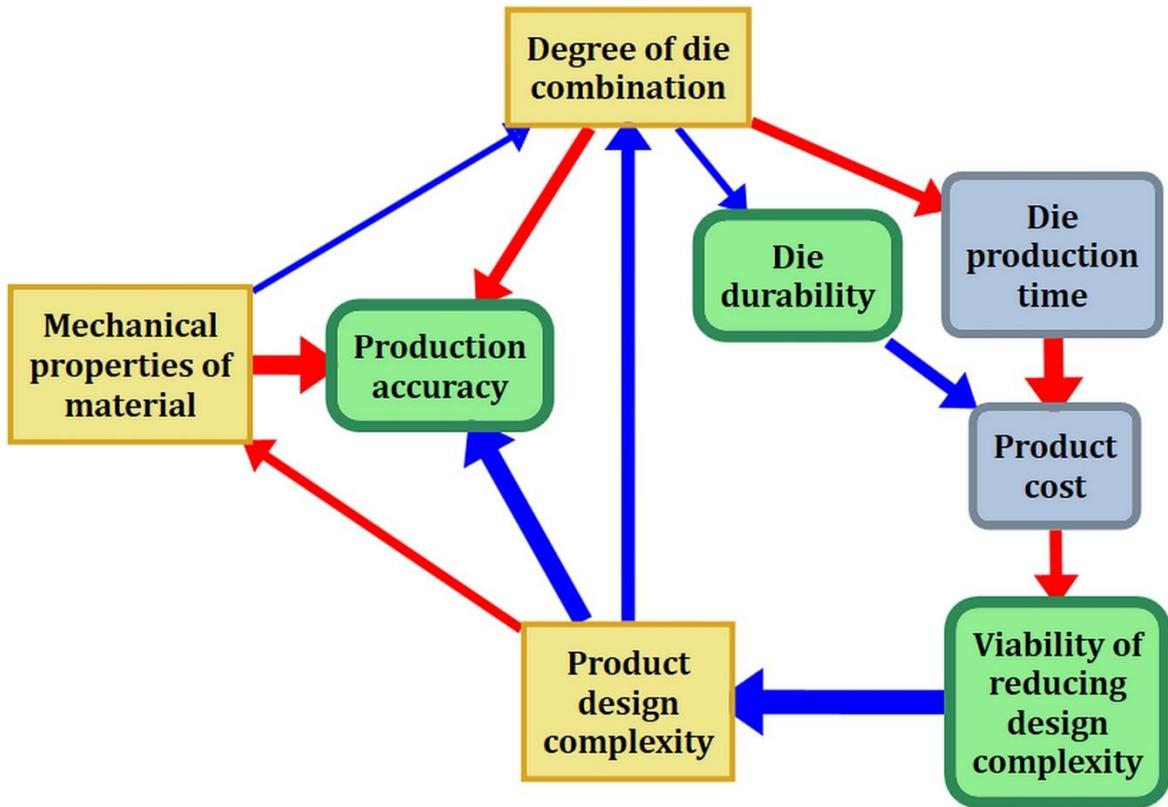


Fig. 3. Metaphor of concept differentiation by type

3. A group of metaphors that allow visualizing FCM system indicators calculated as a result of structure and target analysis. In addition to directly specified cause-and-effect relations, it is necessary to consider all indirect mutual influences of factors in the system in order to perform the structure and target analysis. Transitive closure operation allows that by transforming the initial matrix of influence intensity W into a transitively closed matrix Z , elements of which are pairs (z_{ij}, \bar{z}_{ij}) , where z_{ij} characterizes strength of the positive influence and \bar{z}_{ij} strength of the negative influence of the i -th concept on the j -th one. The algorithm for calculating fuzzy transitive closure is described in detail in monograph [11]. Based on matrix Z , several FCM system indicators can be calculated, a detailed description of which can be found, for example, in [8, 10, 11]. In the example in Fig. 5 visualization objects include comparative intensity and direction of influence of all concepts on the system. The influence

of the i -th concept on the system is calculated by the following formula:

$$\vec{P}_i = \frac{1}{n} \sum_{j=1}^n p_{ij}$$

where n is the number of concepts in the system, p_{ij} is the influence (action) of the i -th concept on the j -th one (i.e. dominating influence intensity between concepts):

$$p_{ij} = \text{sign}(z_{ij} + \bar{z}_{ij}) \max(|z_{ij}|, |\bar{z}_{ij}|), |z_{ij}| \neq |\bar{z}_{ij}|$$

where $\text{sign}(x)$ is a function, returning expression sign x .

Similar to the colours of the arcs, shades of red are used to represent concepts that positively influence the system, and shades of blue represent those negatively affecting the system. Colour saturation determines influence intensity. Thus, “Degree of die combination” and “Die production time” concepts have the strongest positive influence on the system, while “Product design complexity” concept has the strongest negative influence. Meanwhile, white colour means that the concept has no significant influence on the system, which is typical of target concepts that are “stock” ones. In the example under consideration, “Production accuracy” is such a concept.

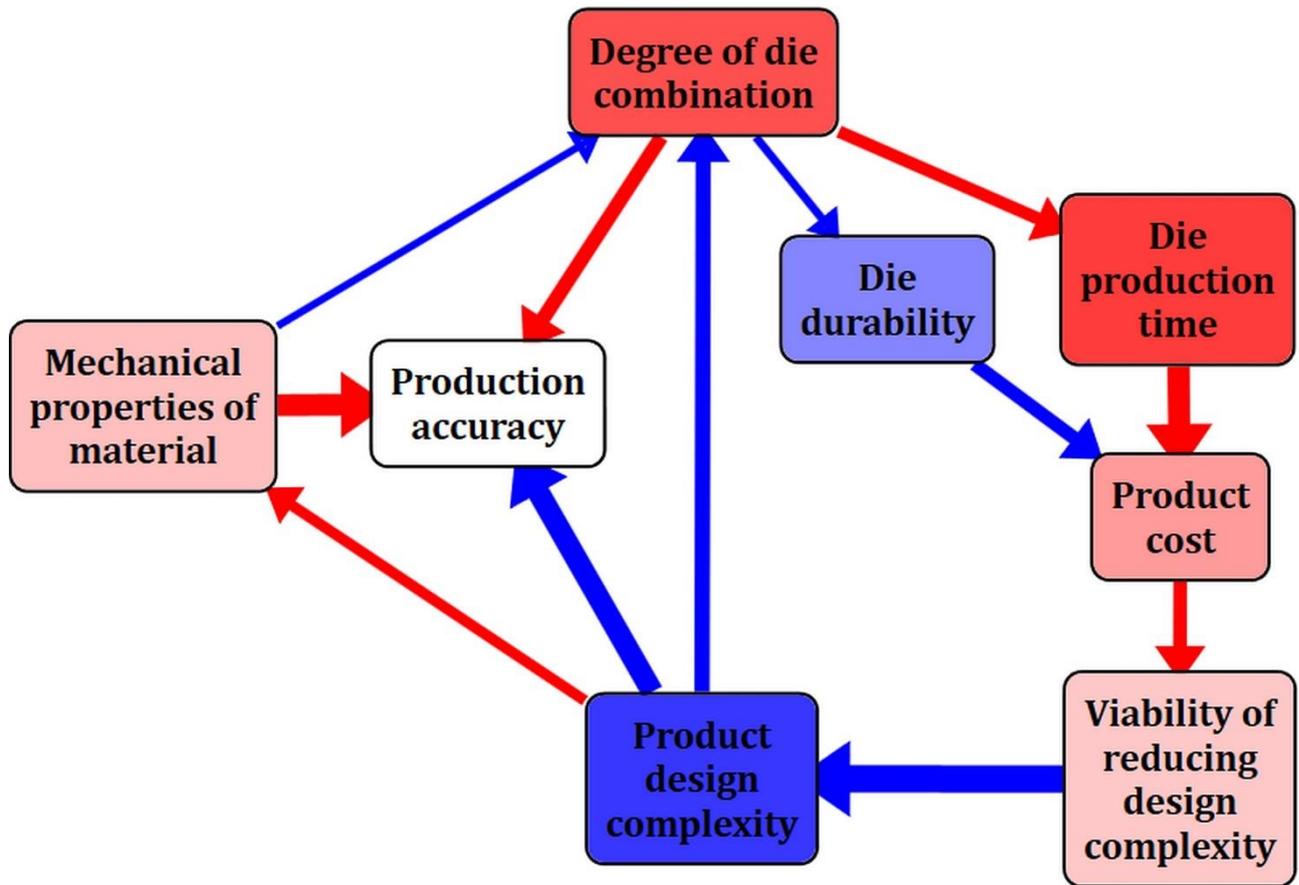


Fig. 4. Metaphor of concept influence on the system

4. “Alpha-cut” metaphor (Fig. 5), allowing to select groups of interrelated concepts characterized by a given cut level relative to the selected system indicator. Distinguishing such groups is possible only for symmetrical indicators. One of such indicators is the indicator of mutual negative influence of concepts, which is calculated by the formula:

$$\vec{n}_{ij} = \vec{n}_{ji} = -S\left(|\bar{z}_{ij}|, |\bar{z}_{ji}|\right),$$

where S is some S -norm operator used to represent the operation of combining fuzzy sets (for example, maximum operator). We will call matrix $\vec{N} = [\vec{n}_{ij}]_{n \times n}$ a cognitive matrix of mutual negative influence. The α -cut of a cognitive map for some symmetric system indicator is a binary relation corresponding to a level set of a fuzzy relation defined based on a cognitive matrix associated with this indicator. So, for the indicator of mutual negative influence and a given cut level α , the corresponding binary relation is derived according to the rule:

$$\vec{n}_{ij}^{\alpha} = \begin{cases} 1, & \text{if } \vec{n}_{ij} \geq \alpha; \\ 0, & \text{otherwise.} \end{cases}$$

For other symmetric system indicators, α -cuts are derived similarly.

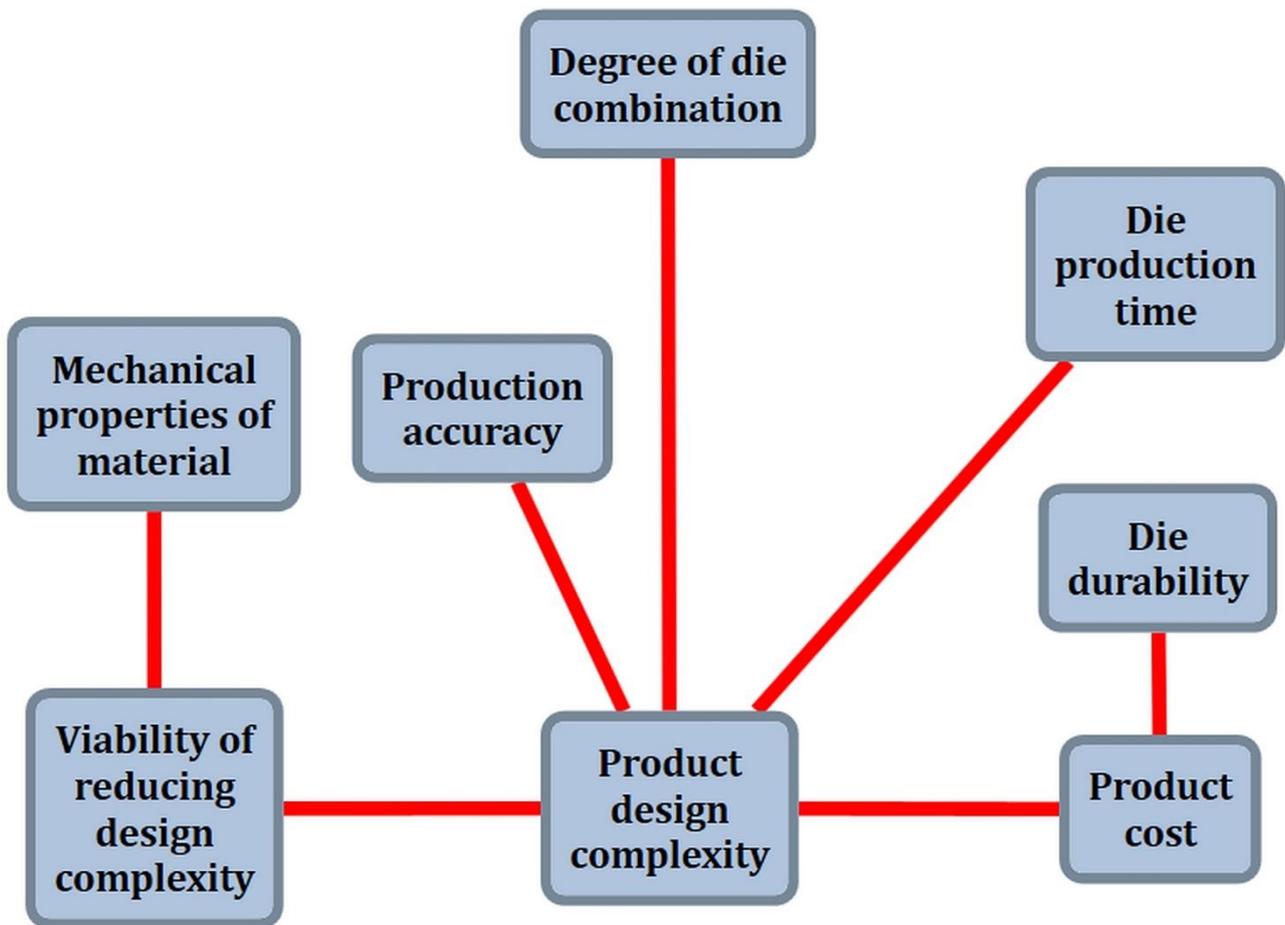


Fig. 5. Alpha-cut metaphor in terms of mutual negative influence indicator

Note that for the alpha-cut, the direction, sign and intensity of the cause-and-effect relation between concepts are not taken into account, so in this case, we use undirected lines of the same color and thickness to represent all the connections.

5. Cognitive clarity criteria

Cognitive clarity of some information is mainly characterized by the ease of intuitive understanding of the corresponding descriptions, messages, etc. Lack of cognitive clarity is manifested in the fact that a person pauses to think, finds difficulties in trying to understand what is said or written. This can be expressed in an observed slowdown of the comprehension process. Another consequence can be the omission of meaningful information that escapes notice [1].

On the other hand, in works dealing with graph visualization, so-called “aesthetic criteria” are often cited, which are associated with the increasing visual clarity of the resulting graph image. A detailed review of

such criteria is given in book [7]. Since FCM, as mentioned above, is a weighted directed graph, it is logical that aesthetic criteria will make a significant contribution to improving visual clarity of its image.

It can be noted that many aesthetic criteria are implicitly aimed at improving cognitive clarity of a graph image, and, conversely, an image that fully possesses the properties of cognitive clarity is most likely to be aesthetically attractive. Thus, it is quite appropriate to include aesthetic criteria with the cognitive clarity criteria, and we can further refer only to this group of criteria.

Thus, the following cognitive clarity criteria are proposed as the basis for constructing FCM visualization metaphor:

- 1) directionality of the arcs: more convenient (hereinafter – all other conditions being equal) for “reading” of FCM are the “top-down” and “left-to-right” directions of the arcs (these directions coincide with the usual reading directions);

- 2) unidirectionality of consecutive arcs image: if it is not required to constantly change gaze direction, then visual comprehension of paths and cycles of a graph will be performed faster;
- 3) minimizing intersections of arcs: ideally they should be absent, and if this is impossible (for a non-planar graph), their number should be minimized;
- 4) minimizing the number of curved arcs: images with straight arcs are more convenient for perception;
- 5) minimizing length of arcs (both the aggregate length and maximum length): the shorter the arcs are, the easier it is to see which concepts are connected to each other, and the more links can be seen simultaneously;
- 6) minimizing scatter in length of arcs: images in which all the arcs are about the same length are more convenient for perception;
- 7) maximizing angles between arcs incident with one point: at small angles between such arcs, they will “merge” with each other near the points, which can hinder visual determination of their directions;
- 8) optimizing location area: for space efficiency, graph should be placed in a rectangular area, the format of which (i.e. aspect ratio) corresponds to the current format of a graphic area of visualization subsystem;
- 9) emphasizing graph symmetry: images symmetrical about a certain axis or center are more convenient for perception and analysis.

Thus for FCM visualization metaphor described above, we conclude that its second most important component (after graph visualization algorithms) should be the introduced criteria of cognitive clarity. Analyzing these criteria, we can conclude that many of them contradict each other, and it is usually impossible to ensure that the image conforms to all the criteria from the algorithmic point of view. Thus, regardless of specific features of FCM visualization metaphor implementation, it is necessary to develop decision rules that simulate various forms of compromise between criteria.

6. Interactive control of fuzzy cognitive map visual representation

In [4], it is noted that interactive control of visual image ensures user’s direct participation in manipulating the image and forms the basis for in-depth data analysis. In addition, interactive model control system is one of the ways to verify the obtained solutions and therefore it ensures accelerated achievement of analysis goal in the case when this method is convenient for the user. Thus, the interactivity of the visual model becomes a condition for its high performance.

The mechanism of interactive control of the FCM visual representation by a cognitive model in IGLA system provides the following features:

- 1) editing cognitive model structure by editing its visual image;
- 2) restructuring of spatial metaphor taking into account the chosen algorithm for graph visualization;
- 3) transformation of a cognitive model image;
- 4) switching between representation metaphors and adjustment of individual metaphors.

Editing model structure implies the possibility to add, delete, and change concepts and relations by manipulating elements of graphical interface and visual image of the model itself.

Transformation of the image affects neither the structure of the model nor the currently used representation metaphor, however, it allows flexibility to change properties of the final displayed image (thus performing a kind of post-processing). Possibilities of transformation include: smooth image scaling; image rotation at a given angle clockwise or counterclockwise; image mirroring relative to the horizontal or vertical axis; image compressing and spreading along selected directions (Fig. 6).

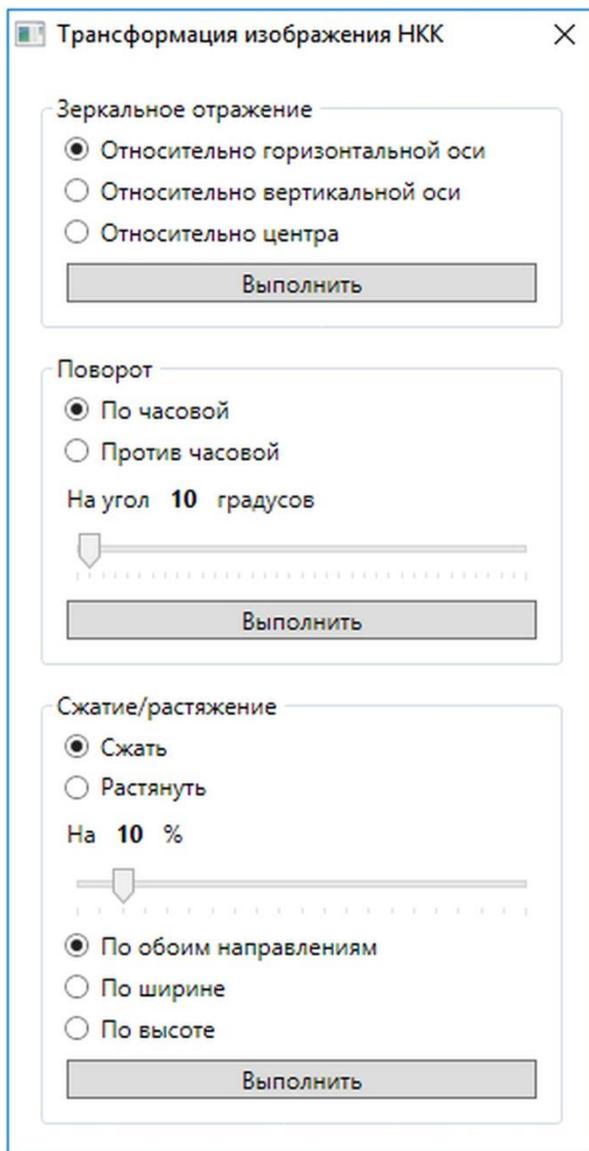


Fig. 6. User interface of FCM image transformation in IGLA system

Finally, due to the possibility to switch between representation metaphors, the researcher can direct his attention at any time to the aspects of the model that are of greatest interest to him at the current stage of the analysis. Besides, functions of adjusting visual characteristics of representation metaphors also contribute to increasing flexibility of this tool and its convenience for the researcher (Fig. 7).

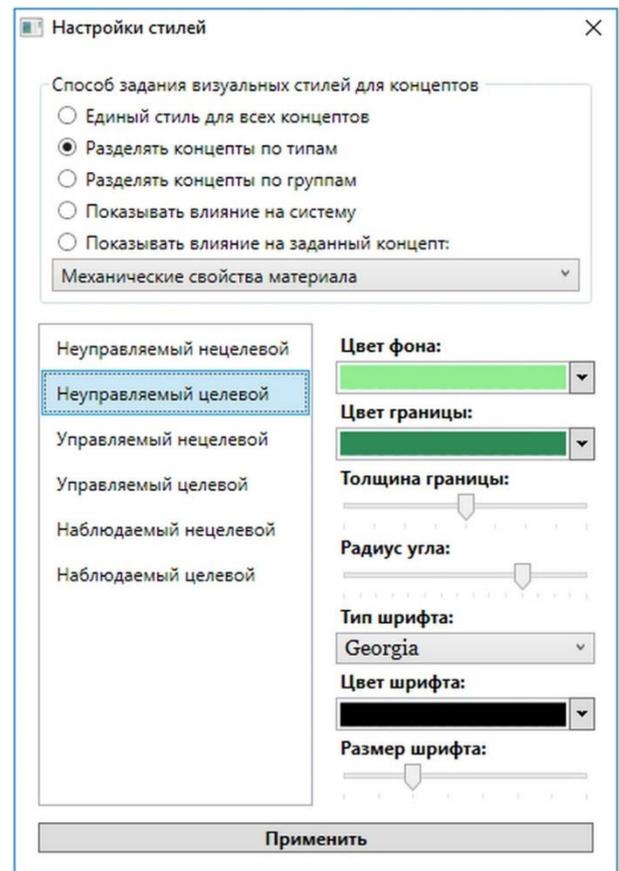


Fig.7. User interface for selecting and adjusting representation metaphors in IGLA system

7. Conclusion

The paper presents a visualization metaphor of fuzzy cognitive maps. Two components of the visualization metaphor are defined – spatial metaphor and representation metaphor. The concept of cognitive clarity is considered. As follows from the analysis, there is a link between the quality of the FCM visualization metaphor and the level of cognitive clarity of the obtained visual image: the higher the level of cognitive clarity provided by the visualization metaphor is, the simpler the process of expert understanding of a cognitive model is when visually analyzing it. To assess the level of cognitive clarity, a set of criteria has been proposed.

Thus, along with graph visualization algorithms, indicators of cognitive clarity form the basis for constructing a FCM visualization metaphor and at the same time they are the most natural quality assessment tool for a constructed metaphor.

We have studied the possibilities of interactive control of FCM visual representation in IGLA system providing flexible adjustment of representation metaphor visual characteristics. They allow the researcher's focusing on the aspects most relevant at a particular stage of analysis.

The possibilities of using visualization metaphors and interactive control of FCM visual representation in IGLA system are exemplified by a cognitive model of decision support in production tooling design [8].

Let us consider directions for further research:

The first one is formalization of the criteria of cognitive clarity described above and the development of a quality assessing method for the FCM visualization metaphor based on formalized criteria, with the implementation of this method in the visualization subsystem of IGLA system.

The second one improving the mechanism of interactive control of FCM visual image in IGLA system, in particular, implementation of the function of optimal spatial metaphor automatic selection with regard to the priorities indicated by the user according to the cognitive clarity criteria.

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